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CURRICULAR INTEGRATION OF MATHEMATICS AND DANCE TO IMPROVE GEOMETRIC REASONING IN SECONDARY SCHOOL STUDENTS

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ABSTRACT

Objective: The objective of this study is to investigate the effectiveness of integrating dance into mathematics education, with the purpose of improving geometric reasoning among secondary school students in Colombia and increasing their interest in STEAM disciplines.

Theoretical Framework: This section presents the main concepts and theories supporting the research. Notable among these are theories of geometric reasoning, such as the Van Hiele model, as well as theories of interdisciplinary education in STEM and STEAM. These theories provide a solid foundation for understanding how integrating artistic disciplines, such as dance, can influence mathematics learning and activate students' creativity.

Method: The methodology adopted for this research includes a pre-post test design using the Van Hiele Test to measure geometric reasoning. The sample consisted of three groups of secondary school students in Colombia. A curricular integration proposal linking dance with mathematics was implemented, and results were evaluated before and after the intervention to determine its impact on geometric reasoning.

Results and Discussion: The results revealed significant improvements in students' geometric reasoning following the intervention, with notable gains in visualization and recognition levels according to the Van Hiele Test. In the discussion section, these results are contextualized within the theoretical framework, highlighting how integrating dance into mathematics teaching can overcome fragmented knowledge and increase students' interest in STEAM disciplines. Possible study limitations, such as sample size and intervention duration, are also considered.

Research Implications: The practical and theoretical implications of this research are discussed, providing information on how the results can be applied to educational practices to improve performance in STEAM disciplines. These implications could include reformulating school curricula to incorporate interdisciplinary approaches that integrate arts and sciences, particularly in the context of secondary education in Colombia.

Originality/Value: This study contributes to the literature by demonstrating the effectiveness of an innovative proposal that integrates dance into mathematics education to improve geometric reasoning. The relevance and value of this research are evidenced by its potential to transform the way STEAM disciplines are taught, offering a more creative and interdisciplinary approach that can motivate more students to pursue careers in science, technology, engineering, art, and mathematics.

Keywords: Mathematics, Dance, Curricular Integration, STEAM Education, Geometric Reasoning.

INTEGRAÇÃO CURRICULAR DE MATEMÁTICA E DANÇA PARA MELHORAR O RACIOCÍNIO GEOMÉTRICO EM ESTUDANTES DO ENSINO MÉDIO

RESUMO

Objetivo: O objetivo deste estudo é investigar a eficácia da integração da dança na educação matemática, com o propósito de melhorar o raciocínio geométrico entre estudantes do ensino médio na Colômbia e aumentar seu interesse nas disciplinas STEAM.

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Referencial Teórico: Esta seção apresenta os principais conceitos e teorias que sustentam a pesquisa. Destacam-se as teorias do raciocínio geométrico, como o modelo de Van Hiele, bem como as teorias da educação interdisciplinar em STEM e STEAM. Essas teorias fornecem uma base sólida para compreender como a integração de disciplinas artísticas, como a dança, pode influenciar o aprendizado da matemática e ativar a criatividade dos alunos.

Método: A metodologia adotada para esta pesquisa inclui um desenho pré-pós-teste usando o Teste de Van Hiele para medir o raciocínio geométrico. A amostra consistiu em três grupos de estudantes do ensino médio na Colômbia. Foi implementada uma proposta de integração curricular que vinculava a dança à matemática, e os resultados foram avaliados antes e depois da intervenção para determinar seu impacto no raciocínio geométrico.

Resultados e Discussão: Os resultados revelaram melhorias significativas no raciocínio geométrico dos alunos após a intervenção, com ganhos notáveis nos níveis de visualização e reconhecimento de acordo com o Teste de Van Hiele. Na seção de discussão, esses resultados são contextualizados dentro do referencial teórico, destacando como a integração da dança no ensino de matemática pode superar o conhecimento fragmentado e aumentar o interesse dos alunos nas disciplinas STEAM. Possíveis limitações do estudo, como o tamanho da amostra e a duração da intervenção, também são consideradas.

Implicações da Pesquisa: As implicações práticas e teóricas desta pesquisa são discutidas, fornecendo informações sobre como os resultados podem ser aplicados às práticas educacionais para melhorar o desempenho nas disciplinas STEAM. Essas implicações podem incluir a reformulação dos currículos escolares para incorporar abordagens interdisciplinares que integrem artes e ciências, particularmente no contexto da educação secundária na Colômbia.

Originalidade/Valor: Este estudo contribui para a literatura ao demonstrar a eficácia de uma proposta inovadora que integra a dança na educação matemática para melhorar o raciocínio geométrico. A relevância e o valor desta pesquisa são evidenciados por seu potencial de transformar a maneira como as disciplinas STEAM são ensinadas, oferecendo uma abordagem mais criativa e interdisciplinar que pode motivar mais alunos a seguir carreiras em ciência, tecnologia, engenharia, arte e matemática.

Palavras-chave: Matemática, Dança, Integração Curricular, Educação STEAM, Raciocínio Geométrico.

INTEGRACIÓN CURRICULAR DE MATEMÁTICAS Y DANZA PARA MEJORAR EL RAZONAMIENTO GEOMÉTRICO EN ESTUDIANTES DE SECUNDARIA

RESUMEN

Objetivo: El objetivo de este estudio es investigar la efectividad de la integración del baile en la enseñanza de las matemáticas, con el propósito de mejorar el razonamiento geométrico entre estudiantes de secundaria en Colombia y aumentar su interés en las disciplinas STEAM.

Marco Teórico: En este apartado se presentan los principales conceptos y teorías que sustentan la investigación. Destacan las teorías del razonamiento geométrico, como el modelo de Van Hiele, así como las teorías de la educación interdisciplinaria en STEM y STEAM. Estas teorías proporcionan una base sólida para comprender cómo la integración de disciplinas artísticas, como la danza, puede influir en el aprendizaje de las matemáticas y activar la creatividad de los estudiantes.

Método: La metodología adoptada para esta investigación comprende un diseño pre-post test utilizando el Test de Van Hiele para medir el razonamiento geométrico. La muestra estuvo conformada por tres grupos de estudiantes de secundaria en Colombia. Se implementó una propuesta de integración curricular que vinculaba la danza con las matemáticas, y se evaluaron los resultados antes y después de la intervención para determinar el impacto en el razonamiento geométrico.

Resultados y Discusión: Los resultados obtenidos revelaron mejoras significativas en el razonamiento geométrico de los estudiantes tras la intervención, con incrementos notables en los niveles de visualización y reconocimiento según el Test de Van Hiele. En la sección de discusión, estos resultados se contextualizan a la luz del marco teórico, destacando cómo la integración de la danza en la enseñanza de las matemáticas puede superar el conocimiento fragmentado y aumentar el interés de los estudiantes en las disciplinas STEAM. También se consideran posibles limitaciones del estudio, como el tamaño de la muestra y la duración de la intervención.

Implicaciones de la investigación: Se discuten las implicaciones prácticas y teóricas de esta investigación, proporcionando información sobre cómo los resultados pueden aplicarse en las prácticas educativas para mejorar el rendimiento en disciplinas STEAM. Estas implicaciones podrían abarcar la reformulación de currículos escolares para incluir enfoques interdisciplinarios que integren las artes y las ciencias, particularmente en el contexto de la educación secundaria en Colombia.

Originalidad/Valor: Este estudio contribuye a la literatura al demostrar la efectividad de una propuesta innovadora que integra la danza en la enseñanza de las matemáticas para mejorar el razonamiento geométrico. La relevancia y valor de esta investigación se evidencian en su potencial para transformar la forma en que se enseñan las disciplinas STEAM, ofreciendo un enfoque más creativo e interdisciplinario que puede motivar a más estudiantes a seguir carreras relacionadas con la ciencia, tecnología, ingeniería, arte y matemáticas.

Palabras clave: Matemáticas, Danza, Integra ción Curricular, Educación STEAM, Razonamiento Geométrico.

1 INTRODUCTION

Education, as a fundamental pillar in the development of society, faces various challenges impacting the quality and interest of students in certain disciplines. The areas of Science, Technology, Engineering, and Mathematics (STEM) and its extension to STEAM, incorporating the Arts, have generated growing concern among academic and scientific communities. The low performance and lack of motivation towards mathematics, with its implications for education and the reduction of professionals in essential careers requiring strong math skills, have garnered particular attention (Videla et al., 2021).

Demotivation towards mathematics is a persistent challenge in global education. The complexity of mathematics and traditional pedagogical approaches that may disconnect from students' daily realities create barriers to engagement and appreciation. This situation has become a central point of interest in STEAM programs worldwide. STEM and STEAM education aim to foster deep understanding of academic disciplines and their practical applications in everyday life. However, demotivation towards mathematics remains an obstacle to achieving these goals (Aguilera & Ortiz-Revilla, 2021).

This concern has led to a reevaluation of teaching methods and the design of innovative pedagogical strategies to make mathematics more accessible and engaging. STEAM programs recognize that interdisciplinary connections improve conceptual understanding, stimulate creativity, and enhance intrinsic motivation. Including the Arts in the STEM paradigm reflects a need to address demotivation through multidisciplinary approaches, with art providing a more creative and visual dimension to mathematics (Yığ, 2022; Ertas, 2022).

This study investigates the effects on motivation and mathematical performance of secondary students by integrating real-world and sociocultural contexts through the application of mathematical thinking in folk dance. The hypothesis is that bodily involvement and recognizing math applications in dance can increase interest and positive attitudes towards problem-solving and geometric modeling. The proposed integrated curriculum connects various disciplines through thematic units, making learning more relevant and engaging for students (Yığ, 2022). Integrated curricula involve synthesized learning through thematic areas and experiences designed to reinforce each other, developing the child's ability to transfer learning to other environments.

1.1 THE PURPOSE OF AN INTEGRATED CURRICULUM

Key indicators of national and international student performance highlight the need for educational improvements, revealing deficiencies in math and science achievement. The widespread belief that public schools are not adequately motivating or preparing students for STEM activities underscores this need (Falloon et al., 2020). Interdisciplinary proposals, such as integrated curricula, are essential for increasing student participation, interest, and performance in STEM and STEAM areas (Juca-Aulestia et al., 2021). Despite challenges in public educational settings, such as limited resources and administrative demands, integrating disciplines like dance and mathematics is crucial for creating a more engaging and effective learning environment.

According to Ertas (2022), knowledge today is increasingly interdisciplinary and integrated, requiring similar learning approaches in public schools. Integrated curricula engage students and deepen their understanding of content, with notable achievements in hands-on activities and reduced performance gaps, especially in science and mathematics (Ertas, 2022). The primary purpose of an integrated curriculum is to have a student-centered approach that engages and enhances learning while increasing interest. Higher-order thinking skills, cooperative learning, and consideration of others' values are emphasized. Students gain a deeper understanding of proposed learnings, preparing them for lifelong learning by linking classroom experiences with the real world (Bautista et al., 2016; Kneen et al., 2020; Norman & Wall, 2020).

In contemporary educational exploration, curriculum integration has emerged as an essential pedagogical strategy that seeks to harmonize various areas of knowledge to foster a more holistic and cohesive understanding of the world. Instead of considering disciplines in isolation and fragmentation, curriculum integration focuses on their interrelation, revealing how they can complement and enrich each other (Kneen et al., 2020).

Curriculum integration contributes to developing critical and creative thinking skills. In traditional education, where subjects are studied in isolation, students may not have the opportunity to transfer or adapt what they have learned to different contexts (Muirhead et al., 2022). Curriculum integration fosters this type of adaptability and mental flexibility, requiring students to consider problems and concepts from multiple angles and perspectives, enriching their reasoning ability and strengthening their conceptual understanding. On the other hand, curriculum integration also represents a challenge for pedagogy and educational structure. Successful implementation requires effective collaboration and communication among teachers, sharing ideas and strategies for content integration. Moreover, this may necessitate significant changes in how curricula are structured and students are assessed. Some institutions have addressed this challenge by creating interdisciplinary teaching teams or adopting project-based learning approaches that allow for a more natural exploration of topics across multiple disciplines (Roehrig et al., 2021).

As our understanding of how students learn best has advanced, curriculum integration models have evolved to adapt to these new perspectives. This initially involved seeking superficial connections and creating thematic links between subjects. However, as pedagogy has evolved, curriculum integration approaches have become more sophisticated and ambitious (Norman & Wall, 2020). Interdisciplinary models have emerged as a response to the need for deeper and more meaningful integration. These models seek to connect disciplines around common themes or problems, allowing students to explore and understand the complexity of real-world phenomena (Muirhead et al., 2022). Instead of studying each discipline in isolation, students engage in projects and activities that demand the application of knowledge and skills from multiple fields.

Conversely, transdisciplinary models have taken curriculum integration to a higher level, transcending the boundaries of individual disciplines. These models address complex, authentic problems from multiple perspectives, recognizing that reality is not limited to established disciplinary boundaries (Kneen et al., 2020; Norman & Wall, 2020). Instead of compartmentalizing knowledge, transdisciplinary models emphasize the interconnection and interdependence of disciplines, inviting students to adopt a holistic approach in their learning and problem-solving.

Thus, curriculum integration models have evolved from mere correlation of themes to deeper and more holistic approaches. Interdisciplinary and transdisciplinary models promote more meaningful and coherent integration of disciplines, allowing students to address real-world problems from a broader and more complex perspective (Kneen et al., 2020). These approaches reflect a more mature understanding of how students learn and how their educational experience can be enriched through curriculum integration.

1.2 EXPLORATION OF KNOWLEDGE CONSTRUCTION IN DANCE AND MATHEMATICS

Exploring the ways knowledge is constructed in dance and mathematics reveals fascinating characteristics and processes inherent to each discipline. Mathematics, as a formal science, focuses on the study of structures, quantities, and spaces through an abstract and logical language. Conversely, dance manifests as an art form where body movement serves to express emotions and ideas, often within a musical and cultural context (Belcastro & Schaffer, 2011).

Mathematics, with its rigorous analytical approach, has been an indispensable tool for unraveling the mysteries of the universe and providing solutions to complex problems. Mathematical abstraction is key in this process, allowing for the representation of structures and relationships in a generalized manner. Through theorems, equations, and models, mathematics has shaped countless scientific discoveries and technological advancements (Tanswell & Kidd, 2021). Additionally, its universal nature makes the mathematical language understandable and applicable across various contexts and cultures, serving as a bridge between disciplines.

Dance delves into a different spectrum of human expression. Through bodily movement, dance narrates stories, conveys emotions, and celebrates cultural traditions. Unlike the abstract nature of mathematics, dance is concrete and physical (Ramadanova & Kulbekova, 2023). In dance, the body becomes an instrument that, in harmony with music, communicates messages and evokes emotional responses in the viewer. Moreover, dance has the ability to cross linguistic barriers, allowing people from different cultural backgrounds to connect and share common experiences.

When approached together, mathematics and dance can complement each other in surprising ways. The precision and structure of mathematics can be used to analyze and enhance elements of choreography and dance performance. Similarly, the physical expression of dance can be useful for visualizing and understanding abstract mathematical concepts. For example, dance can graphically represent the relationship between geometric shapes and movements, or model mathematical patterns through dance sequences. Thus, the integration of dance and mathematics offers a unique and enriching perspective for constructing knowledge in both fields.

One element both disciplines share is the use of space. Dancers move and create shapes and figures in space, while mathematics, especially geometry, studies the properties and dimensions of space. By observing a dance choreography, one can identify geometric patterns and symmetries in the movements and spatial arrangements of the dancers. Additionally, time and rhythm represent another point of convergence. In dance, rhythm guides movements and becomes a temporal structure framing the execution. Mathematics, in turn, analyzes rhythm through patterns and sequences and is fundamental in musical theory (An et al., 2019; Belcastro & Schaffer, 2011; López et al., 2022).

1.3 IDENTIFICATION OF SIMILARITIES AND DIFFERENCES IN THINKING AND REASONING PROCESSES IN BOTH DISCIPLINES

In the intricate web of human knowledge acquisition, two seemingly disparate domains, dance and mathematics, converge and diverge in various ways. At first glance, dance, with its focus on emotional expression and physical grace, and mathematics, with its rigorous logic and abstract structures, may seem worlds apart. However, delving into the cognitive processes underlying each discipline reveals a network of fascinating similarities and contrasts (Mattingly, 2022; Moerman, 2013; Shamir et al., 2019). This comparative analysis reveals how thinking and reasoning patterns can be transferable and applicable across different fields of knowledge.

In discerning the similarities and differences in thinking and reasoning processes between dance and mathematics, it is imperative to consider the epistemological foundations of both disciplines (Warburton, 2011). Mathematics, characterized by its precision and rigor, employs deductive logic and often starts from axioms and theorems to develop more complex structures. Mathematical reasoning involves a high level of abstraction and systematic structuring of concepts. In contrast, dance, as an art form, tends to be more intuitive and emotional in its approach. However, it is crucial to recognize that dance also possesses structure and patterns, although these are more fluid and often less rigid than in mathematics (Mattingly, 2022).

Beyond epistemological foundations, it is valuable to examine the role of tools and techniques in constructing knowledge in both disciplines. In mathematics, instruments like calculators, modeling software, and simulation techniques are crucial in problem-solving and complex analysis. In dance, the human body acts as the primary tool, and elements like music, costumes, and scenery contribute to the construction and communication of meanings through movement.

Another revealing aspect is the collaborative approach in both disciplines. In mathematics, collaboration can take the form of joint research and problem-solving, while in dance, collaboration is often physical and spatial (Vogelstein et al., 2019). In a dance setting, dancers must be aware of their own movements and those of others, requiring a high degree of synchronization and non-verbal communication. This can be seen as a form of real-time problem-solving, where each dancer contributes to constructing a cohesive piece through their movement and presence on stage.

Lastly, it is relevant to highlight the role of tradition and innovation in dance and mathematics. In mathematics, there is a deep tradition of theories and concepts built over centuries, with modern advances often based on these historical foundations (Nurjannah & Wijaya, 2022). In dance, tradition manifests in styles and techniques passed down through generations. However, both dance and mathematics require innovation for the discipline's development. In dance, this can involve exploring new movements or merging different styles, while in mathematics, innovation can involve developing new theories or applying mathematical concepts in emerging fields.

Observing the creative process reveals that both dance and mathematics involve a level of inventiveness and exploration. In mathematics, theorems and formulas often arise from identifying patterns and seeking logical consistency. In dance, creating choreography involves identifying movement patterns and exploring expression and interpretation. In this sense, one might argue that there is a parallel between theorem formulation in mathematics and choreography creation in dance, though the former is rooted more in logic and the latter in expression (Horsthemke, 2016). Delving into the nature of the creative process, we see that in mathematics, creativity manifests in the ability to formulate and test conjectures and find novel solutions to problems. This creativity is intrinsically linked to abstraction and logical thinking. A mathematician may find beauty in the elegance of a proof or the symmetry of an equation, which also relates to an aesthetic sense, though different from that traditionally associated with the arts (Swaminathan & Schellenberg, 2015a).

Despite the differences in the nature of creativity in both disciplines, there are points of convergence where they intertwine. For example, structuring a choreography in dance may require a systematic approach similar to problem-solving in mathematics. Likewise, in mathematics, moments of intuition and artistic vision can guide the discovery process, similar to how a choreographer might be driven by an aesthetic vision. Ultimately, both dance and mathematics are forms of language and communication that, through different means, seek to discover, interpret, and convey aspects of our world and human experience (Leandro et al., 2018). However, the fundamental divergence lies in the nature of the results and the subjectivity inherent in their interpretation. While in mathematics, the validity of a result is judged by objective criteria of consistency and logic, in dance, the appreciation and impact of a piece are subjective and can vary among different spectators.

1.4 RESEARCH QUESTIONS

The present study aims at identifying the results that a curriculum integration proposal between school mathematics and artistic dance education have at the secondary education level in the Colombian context. The following research questions are investigated:

- What epistemological and didactic relationship exists between mathematics and dance?
- What are the convergence points in a curricular proposal of integration between maths and dance?
- What results are observed in the participating children regarding their levels of geometric reasoning?

2 MATERIALS AND METHODS

2.1 POPULATION AND SAMPLES

The study population consisted of 83 secondary school students and 15 teachers from the two disciplines: dance and mathematics, not necessarily involved in school contexts.

Students: Eighty-three students were selected through a purposive sample and were distributed as follows: 33 students formed the experimental mathematics group, 18 students formed the experimental dance group, and 32 students formed the control group (students in the mathematics class who did not apply the curriculum integration proposal and studied the subject's topics in the traditional way). Students belong to a public school in Bogotá, Colombia, which serves a vulnerable population from socioeconomic strata 0, 1, and 2. The ages of the students in this academic space range between 12 and 16 years.

Teachers: To promote rigor in this research process, the study population was identified using an inquiry tool called actor mapping, which allows for the identification of actors interacting in the development of the research. In this case, it includes teachers and professionals from both disciplines according to their degree of interest and experience in integration processes. A discussion forum was publicly convened, involving 15 professionals and teachers from various Latin American countries interested in sharing their interests and experiences in both disciplines and in integration proposals in the school context. The research was supported by the IESMART School of Educational Innovation, which has a network of over 1200 teachers and professionals in STEAM areas and provided this space for the forum's development.

2.2 INSTRUMENT: GEOMETRIC REASONING

To analyze this variable, a pre-test and post-test will be applied to determine the students' reasoning level at the beginning of the course and at the end of the curriculum innovation application, using the Van Hiele Geometry Test developed at the University of Chicago by the CDASSG project (Cognitive Development and Achievement in Secondary School Geometry).

This instrument contains 25 items, structured into 5 groups of 5 items each, organized in increasing order of difficulty, one for each of the Van Hiele levels. The instrument items are multiple-choice, with the student choosing one of five possible answers (from A to E), with only one correct answer. The instrument has two parts: a booklet containing the general instructions and the 25 questions, on which the student should not mark answers or make any drawings or marks; and an answer sheet for the 25 items with the 5 response options for each (A-B-C-D-E) for the student to choose the correct one. The answer sheet also provides space for the student to make drawings and calculations.

3 RESULTS

3.1 EPISTEMOLOGICAL AND DIDACTIC RELATIONSHIP EXISTS BETWEEN MATHEMATICS AND DANCE

Integrating seemingly disparate disciplines such as mathematics and dance can challenge academic conventions but also opens a world of possibilities for interdisciplinary exploration and knowledge expansion. In this section of the results chapter, we delve into the search for epistemological and didactic convergence points between two seemingly divergent fields: mathematics, with its logical and abstract rigor, and dance, with its physical and emotional expression. Through detailed analysis, we explore how these two areas of knowledge can find common ground, not only in content and methodology but also in terms of the benefits they can offer students. In this context, we will examine the observations, reflections, and conclusions derived from our research, with the aim of shedding light on the possibilities and challenges of integrating mathematics and dance in the educational realm.

Epistemological convergence refers to the intersection or connection between different fields of knowledge in terms of their epistemological foundations, that is, their principles, methods, and ways of knowing and understanding the world. It involves identifying how different disciplines share certain epistemological bases or how they can be integrated to enrich the understanding of complex phenomena from diverse perspectives.

In the context of education, epistemological convergence implies recognizing how different areas of knowledge, such as mathematics and dance, can have common points in their approach to understanding and constructing knowledge. For example, both disciplines can share principles of abstraction, analysis, symbols, and representations, although applied differently in each field. By identifying the epistemological convergence between disciplines, opportunities can be found to integrate them more effectively in teaching and learning, promoting a more holistic and profound understanding of the concepts and phenomena studied. This can enrich the educational experience by fostering interdisciplinarity and the connection between seemingly distinct areas of knowledge.

Table 1

Epistemological Convergence Points

Mathematics	Dance	Convergence	Key Insights
Study of patterns and structures	Use of movement, rhythm, and time patterns	Both rely on patterns and structures	Use of Dance to Visualize Mathematical Concepts Exploration of Numerical and Geometric Patterns Through Movement Development of Abstraction and Generalization Skills from Patterns and Structures
Logical and abstract rigor	Physical and emotional expression	Logic and creativity	Creative Problem-Solving in Mathematics Use of Logic in Choreography Critical and Analytical Thinking

Didactic convergence refers to the integration of different pedagogical approaches, teaching strategies, and didactic methodologies from various disciplines or areas of knowledge. It involves identifying how teaching methods and approaches from one discipline can be applied to another to enrich the educational process and promote more meaningful learning. In the case of didactic convergence between mathematics and dance, ways to teach mathematical concepts using methodologies and resources specific to dance, and vice versa, are explored.

By linking mathematics and dance, the aim is to facilitate the understanding and retention of mathematical concepts through bodily and sensory practice. This means that students do not only learn passively through observation or verbal explanation but also actively experience the concepts through movement and bodily expression. This kinesthetic methodology allows for a deeper and more meaningful understanding of mathematical concepts, as students internalize them through practical experience.

Moreover, didactic convergence between mathematics and dance involves identifying and using pedagogical strategies that promote the development of cognitive and socio-emotional skills. This includes problem-solving, collaboration, and communication, where students work together to solve mathematical challenges through creative exploration and the exchange of ideas. Additionally, the application of design and structure principles is encouraged, where students can create and perform choreographies that reflect mathematical concepts such as patterns, symmetry, proportions, and geometry.

Table 2

Didactic Convergence Points

Mathematics	Dance	Convergence	Key Insights
Kinesthetic learning	Kinesthetic learning through movement	Learning through movement	Kinesthetic Activities for Teaching Mathematical Concepts
Problem-solving	Problem-solving in movement, rhythm, and space	Developing problem-solving skills	Incorporating Dance in Math Teaching
Collaboration and communication	Collaborative creation of choreographies	Teamwork and effective communication	Development of Learning Through Movement
Application of design and structure principles	Visual composition on stage	Use of design and structure principles	Problem-Solving Through Dance Use of Dance to Teach Problem-Solving Strategies Perseverance and Learning from Errors

Exploration of Practices and Perspectives on Curriculum Integration Between Mathematics and Dance in the Latin American Context.

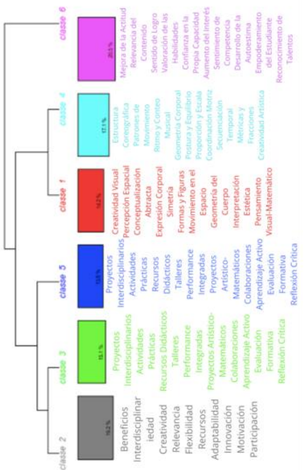
The focus groups were structured to introduce participants to the research approach and curriculum integration proposal, along with findings on the epistemological and didactic convergence between mathematics and dance. Discussions centered on three main topics:

1) feasibility of curriculum integration: participants explored the practicality of combining mathematics and dance in the school curriculum, presenting arguments for and against;
2) integration experiences: analysis of past integration experiences between dance and mathematics, showcasing practices in educational and artistic settings and their impact on learning;
3) relationship between dance and geometric thinking: reflection on how dance practice can enhance geometric and spatial thinking in mathematics.

Using Iramuteq software for lexicometric analysis, six major categories were identified. The first category, "Feasibility of Curriculum Integration Between Dance and Mathematics," highlighted key supportive aspects, including enhanced interdisciplinarity, creativity, and a more comprehensive educational experience. Flexibility, adaptability, and the need for adequate resources and innovation were also emphasized. The potential to motivate students and increase participation was particularly noted. Table 1 shows the results of lexicometric analyses of focus group results.

Table 3

Iramuteq Results of Focus Group Analyses.



Class	Percentage	Themes and Terms
Class 2 (Gray)	19.2%	- Interdisciplinary Benefits - Creativity – Relevance - Flexibility - Resources – Adaptability – Innovation - Motivation - Participation
Class 3 (Green)	15.1%	- Interdisciplinary Projects - Practical Activities - Didactic Resources- Workshops - Integrated Performances - Artistic-Mathematical Projects - Collaborations - Active Learning - Formative Assessment - Critical Reflection
Class 5 (Blue)	13.8%	- Interdisciplinary Projects - Practical Activities - Didactic Resources - Workshops - Integrated Performances - Artistic-Mathematical Projects - Collaborations - Active Learning - Formative Assessment - Critical Reflection
Class 1 (Red)	14.2%	- Visual Creativity - Spatial Perception - Abstract Conceptualization - Body Expression - Symmetry - Forms and Figures - Movement in Space - Body Geometry - Aesthetic Interpretation - Visual-Mathematical Thinking
Class 4 (Cyan)	17.1%	- Choreographic Structure - Movement Patterns - Musical Rhythm and Counting - Body Geometry - Posture and Balance - Proportion and Scale - Motor Coordination - Temporal Sequencing - Metrics and Fractions - Artistic Creativity
Class 6 (Purple)	20.5%	- Attitude Improvement - Content Relevance - Sense of Achievement - Valuing Skills - Self-Confidence - Increased Interest - Sense of Competence - Self-Esteem Development - Student Empowerment - Talent Recognition

The second class on integration experiences covered a wide range of educational approaches and practices. Participants mentioned the implementation of interdisciplinary projects that combine elements from both disciplines to offer a more comprehensive and enriching learning experience. Additionally, the practical activities of students in exploration and creation, and the use of innovative didactic resources to facilitate the understanding of complex concepts were highlighted. Workshops and integrated performances were mentioned as effective strategies to promote collaboration and active learning among students. The importance of artistic-mathematical projects, which allow students to explore mathematical concepts through artistic expression, was also emphasized. Collaborations between dance and mathematics teachers were valued for enriching teaching and learning. Finally, the importance of formative assessment and critical reflection as tools to continuously improve curricular integration practices was emphasized.

The third class explored the close relationship between artistic appreciation and geometric thinking, highlighting how the integration of both disciplines can enrich the learning process. Participants emphasized the importance of visual creativity as a tool to explore mathematical concepts through artistic expression. Furthermore, spatial perception and abstract conceptualization were discussed as fundamental skills in both dance and mathematics, which can be enhanced through integrative practices. Body expression was identified as a means to understand and communicate geometric concepts tangibly and experientially. Symmetry, geometric shapes and figures, as well as movement in space, were recognized as common elements between artistic appreciation and geometric thinking that can be explored and analyzed interdisciplinarily. The importance of aesthetic interpretation and visual-mathematical thinking as tools to deepen the understanding of the relationship between both disciplines and foster a holistic learning approach was also highlighted.

A fourth class resulted from the focus group discussions, highlighting the relationship between mathematics and elements of dance, emphasizing how the integration of both disciplines can enrich the teaching and learning process. Choreographic structure was identified as an element sharing similarities with mathematical patterns, as both involve the meaningful organization and sequencing of movements or numbers. Rhythm and musical counting were recognized as elements that can be linked to mathematical concepts, such as metrics and fractions, requiring an understanding of temporal division and the relationship between musical times. Body geometry and posture were highlighted as fundamental aspects in both dance and mathematics, as both disciplines involve understanding structure and form. Proportion and scale were emphasized as mathematical concepts present in motor coordination and choreographic creation, where precision and harmony are essential. Artistic creativity was identified as a bridge between mathematics and dance, as both disciplines require a creative approach to problem-solving and artistic expression. Overall, these similarities between elements of dance and mathematical concepts suggest a fertile ground for curricular integration and mutual enrichment of both disciplines.

The fifth class focused on student motivation and self-concept. It was observed that improving attitudes towards both disciplines arises from the relevance of the content, which becomes more meaningful when connecting abstract concepts with concrete and artistic experiences. This connection enhances the sense of achievement and skill appreciation, as students perceive tangible progress in their understanding and execution. Confidence in one's abilities is strengthened by experiencing success in solving both mathematical and choreographic problems, contributing to increased interest and a sense of competence.

Additionally, self-esteem develops as students feel empowered to express themselves creatively and recognize their talents in both mathematical and artistic fields. Together, these findings suggest that integrating dance and mathematics can have a positive impact on student motivation and self-concept, fostering greater engagement with learning and a more positive perception of their skills and achievements.

3.2 IMPLEMENTATION OF AN INTEGRATED DIDACTIC DESIGN BETWEEN SCHOOL MATHEMATICS AND ARTISTIC DANCE EDUCATION FOR SECONDARY LEVEL

Based on the results obtained in the first two phases of the research, the design and implementation of the curriculum integration proposal between mathematics and artistic dance education were developed. This involved a literature review and the relationship between theory and practice, incorporating didactic proposals that create spaces for critical reflection and promote the analysis of the school context and the inclusion of the body in learning processes through everyday activities like dance. During its development and execution, opportunities for change and improvement were identified and applied, generating various actions to address the identified limitations.

The construction and implementation were based on the five phases of the Van Hiele model. Ten didactic units were created for two study groups: the experimental mathematics group, consisting of eighth-grade students who implemented the proposal during mathematics classes, and the experimental dance group, consisting of vocational dance students who developed the proposal during extracurricular sessions on weekends.

The ten didactic units cover basic notions of plane geometry, aiming to enhance the development of geometric reasoning and reflect on the role of the moving body in learning processes and the cognitive power of dance. Each activity aims to intensify the kinesthetic load in the mathematics classroom while working on constructing notions and concepts in both learning areas simultaneously.

Additionally, understanding the body as a means of expression and description of knowledge introduces semiokinesis, which can be defined as the semiotics of the body. Semiotics is the science that studies different sign systems that enable communication between individuals, their modes of production, functioning, and reception. Therefore, semiokinesis is defined as the semiotics of the body and movement.

When drawing or tracing the silhouette of one's body and that of peers, or tracing certain geometric shapes after discovering them on the silhouettes of various dancers, students start developing sensitivity towards the semiokinesis of the dancer or even themselves as individuals, questioning how the body expresses and communicates an idea, notion, or thought. The various figures observed from their own corporeality and that of others allow them to recognize themselves as subjects of information and communication through the body. Throughout the activities, there is a continuous discussion about meaning and representation, exploring which choreographic exercises or classroom games foster these two cognitive abilities.

The relationship between semiotics and the history of dance and mathematics lies in the connection between the history of the dancing body and symbols and signs as an essential component in constructing, developing, and communicating mathematical thought. This connection is fundamental in the learner's journey from concrete and manipulable experiences to abstract and formal representation. Approaching mathematics through kinesthetic and proxemic experiences and using the semiotics of the body to analyze the coding and decoding of dances and various choreographic exercises helps regain awareness of different body parts, understanding them as something akin to kino morphemes, analogous to the phonemes and morphemes of everyday language.

Furthermore, from the perspective of multiple intelligences and learning styles, it was possible to identify kinesthetic learners throughout the implementation, recognizing their ability to use the body to tackle different challenges, solve problems, and engage in physical activities not necessarily related to sports, as is common in most institutions. It is known that this type of intelligence manifests naturally in children, but unfortunately, school often blurs or ignores these natural abilities, leading to a lack of kinesthetic experiences that benefit physical and emotional health and overall learning.

Dance practices accompanying each unit include Ballet and Folk Dance genres. In contemporary dance exploration processes, more variables, forms, and actions that could be approached from other geometries and variational thinking are evident, representing an opportunity for generating new questions.Different activities work on enhancing intrinsic perceptual relationships (rhythmic, dynamic, mimetic, structural, textual, qualitative) and extrinsic perceptual relationships (construction of archetypes, psychological, narrative).

Although the focus is on the geometrization and schematization of the body in classical dance, different approaches to Colombian folk dance are also explored, recovering heritage and moving away from viewing the body as rigid and geometric to embracing a diverse body. This helps students view folk dance as contemporary rather than ancient. However, this raises the dilemma between flexibility and rigor, prompting both disciplines to rethink their methodologies.

The didactic units are constructed considering the following components:

- title: designed to generate curiosity in students and creatively and attractively link both knowledge áreas;
- learning objectives: based on the study of mathematics and art education curricular guidelines in Colombia and curricula from both areas, didactic transposition and methodology are applied to address proposed conceptual areas while simultaneously developing similar, complementary, or argumentation and communication-enhancing learnings from both perspectives;
- concept development: for readers who are experts in either dance or mathematics, a theoretical summary of conceptual areas is presented in clear, simple language, ensuring new learnings even for those implementing the proposal;
- Van Hiele model phases: all units follow the model phases strictly, starting with a preliminary information phase based on students' knowledge, followed by phases of directed orientation, explicitization, free orientation, and integration;
- evaluation: all guides describe evaluation instruments used in accordance with the model.

The table below offers a comprehensive summary of the convergence achieved in each of the guides between the disciplines of mathematics and dance, encompassing both the conceptual scope and learning objectives. These findings derive from the detailed analysis of the results obtained in the first objective of this research, clearly identifying how these two fields intertwined and complemented each other in the integration proposal.

Table 4

Convergence Between Mathematics and Dance in the Pedagogical Guides.

TITLE	CONCEPTUAL SCOPE	LEARNING OBJECTIVE
UNIT 1	THE STAGE: A EUCLIDEAN UNIVERSE.Choreographic games of spatial recognition.	Spatio-temporal orientation, planimetric location, and dynamic exploration of the three-dimensional.
UNIT 2	SEGMENTED. Introduction to choreographic improvisation	The body seen as a set of points, lines, straight lines, and line segments.
UNIT 3	MY GEOMETRIZED BODY. Exploration of basic postures in classical ballet and modern dance	The role of the angle and its measurement in the exploration of posture and body awareness of the dancer.

TITLE	CONCEPTUAL SCOPE	LEARNING OBJECTIVE
UNIT 4	RHYTHM AND PRECISION. Parallelism and Perpendicularity in group dances.	Notion of parallelism and perpendicularity in the execution of coordinated group choreographies.
UNIT 5	TRIANGULAR SCHEME. Posture and body demarcation	The triangle as a union of segments in the construction of body awareness and schema.
UNIT 6	POLYGONS. The imaginary square and other polygonal shapes in dance.	The polygon as a finite sequence of consecutive straight segments as a means of expression through the body image.
UNIT 7	THE CIRCUMFERENCE. A look at the characteristics of circular dances and children's rounds.	The circumference as a symbol of union, geometric perfection, and part of human historical and cultural consciousness.
UNIT 8	TRANSLATION AND LOCOMOTION. The importance of counting and choreographic memory for the dancer.	Translation as an isometric transformation in the education of the dancer's movement.
UNIT 9	ROTATION AND TURN. Use of the protractor and the notion of angle in the scenic space.	Rotation and development of the dancer's kinetic potential.
UNIT 10	GEOMETRIC REFLECTION. Laterality and mirror figures.	The dancer's ability in changing fronts and adapting to various stages.

3.3 IMPACT OF DIDACTIC DESIGN ON GEOMETRIC REASONING IN SECONDARY STUDENTS

The van Hiele levels theory, formulated by Pierre M. van Hiele in 1957, addresses geometric understanding, suggesting it parallels understanding in broader mathematics and other subjects. This understanding is exhibited when individuals draw conclusions in new geometric situations based on provided data and relationships, involving the exploration of new contexts.

Van Hiele proposed that geometric understanding develops through logical sequences, seen as syllogisms where each conclusion is the premise for the next. Students may encounter different situations within these sequences:

a) lack of understanding due to unfamiliarity with the topic;
b) inability to perceive logical relationships between statements;
c) failure to accept conclusions despite understanding relationships, indicating incomplete comprehension;
d) achieving understanding does not guarantee the highest comprehension level, as recognizing sequence types requires advanced understanding.

The van Hiele model of mathematical thinking includes a descriptive part, which identifies reasoning levels, and an instructional part, which guides teachers in helping students reach higher reasoning levels. This text focuses on the descriptive aspect, covering three main elements: reasoning levels, their characteristics, and progression.

There are five reasoning levels, as detailed by Afonso (2003):

1) **recognition**: students learn and recognize shapes by name, viewing figures as wholes without focusing on properties;
2) **analysis**: students identify shape properties and understand geometric figures' parts and properties, often generalizing from examples;
3) **classification**: students logically order shapes and relationships, beginning to make deductive inferences and understand definitions;
4) **deduction**: students grasp formal logical reasoning, using postulates, theorems, and multi-step demonstrations to verify statements;
5) **rigor**: students appreciate rigor, performing abstract deductions, and working within different axiomatic systems, including non-Euclidean geometry.

Dina van Hiele designated levels 2 to 5 as aspect, essence, theory understanding, and scientific understanding of geometry. Clements and Battista added a pre-recognition level (level 0) for individuals who recognize shapes but can't identify common forms. Mason noted that learners at this level distinguish basic shapes but not types within a category.

The van Hiele theory highlights several characteristics:

- sequentiality: progression through each preceding level is required;
- adjacency: each level makes explicit what was implicit in the previous one;
- differentiation: each level has its symbols and relationships;
- separation: people at different levels can't understand each other.

Unlike Piaget, van Hiele suggested that instruction can facilitate level progression. The theory does not directly correlate levels with age; teaching and experiences influence reasoning progress. While some research indicates gradual transitions, the van Hiele model remains a practical tool for geometry teachers, which this study adopts.

Geometric reasoning involves drawing conclusions in new situations based on data and relationships within van Hiele's five levels. Usiskin (1982) suggested the theory isn't fully developed for assigning individuals to the highest level, leading this study to focus on the first four levels relevant to an engineering geometry course, using a Modified van Hiele Levels scale (Usiskin, 1982).

To assign a reasoning level, individuals must demonstrate reasoning characteristic of that level and all previous levels, but not subsequent ones. This study expanded criteria to include cases where students meet criteria for levels n and n-1 but not necessarily for levels n-2 or n-3, aligning with Usiskin's dimensions and indicators.

The research adopted a standard pretest-posttest design using the van Hiele Geometry Test developed by CDASSG at the University of Chicago under Zalman Usiskin (1982). The procedure was:

1) informing students about the research objectives, unrelated to regular evaluations;
2) administering the pretest after the first session;
3) administering the posttest after completing the integration proposal;
4) transcribing student responses for processing;
5) determining van Hiele reasoning levels in pretest and posttest per guidelines;
6) using the non-parametric Wilcoxon test for significant differences between pretest and posttest medians.

Each student's level was evaluated using two criteria: correctly answering at least 3 or 4 out of 5 questions. The first criterion minimized data loss. Levels were rated 0 or 1 based on correct answers. Scores increased exponentially from 1 to 16 for levels 1 to 5. The weighted sum of individual evaluations determined each student's van Hiele level, with some combinations recorded as "Unclassified" if criteria were not met.Table 7 presents the weighted sums corresponding to each level and the unclassified cases.

Table 5

Weighted sums by van Hiele level.

Level	Weighted Sum
0	0, 2, 4, 8, 16, 18, 20, 24
1	1, 5, 9, 17, 21, 25
2	3, 11, 19, 27
3	6, 7, 22, 23
4	13, 14, 15, 29, 30, 31
U.C.	10, 12, 26, 28

Applying the indicated procedure, the van Hiele level of each student was determined in the pretest and posttest.

3.4 PRETEST RESULTS BY GROUPS

As shown in Table 8, in the pretest, 30% of the students in the experimental mathematics group, which corresponds to a total of 33 students, are at level 0 and 36% at level 1 of reasoning.

At level 2, 33% of the students in this group, which is 11 students, are located. In the experimental dance group with 18 students, 50% are at level 0 and 27.75% at level 1 of reasoning. The remaining 22% can be located at level 2. Finally, in the control group, 31.2% are at level 0 of reasoning, 50% at level 1, and only 18.75% are at level 2. It was also possible to locate all participants in one of the van Hiele levels in the pretest.

Table 6

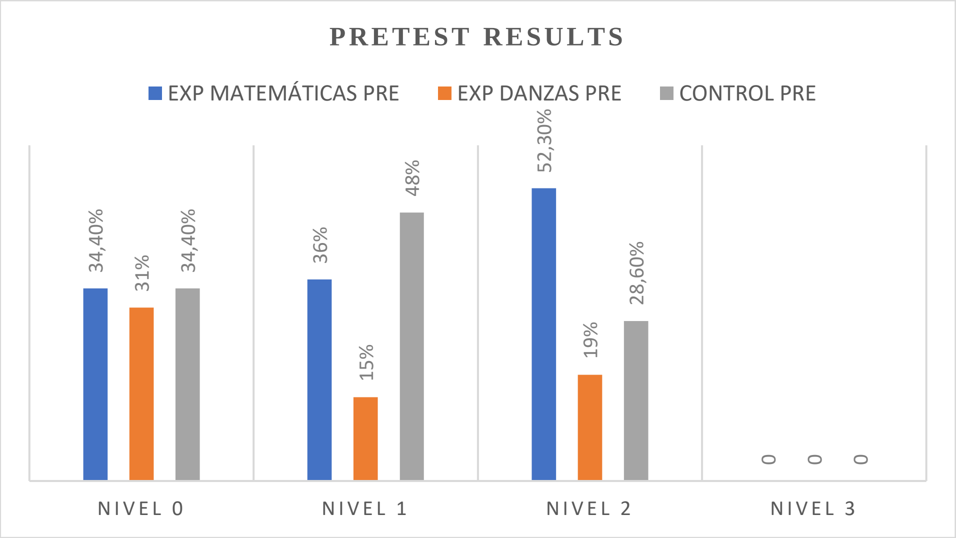
Pretest Results by Groups.

GRUPO	NIVEL PRET	Frequency	Percent	Valid Percent	Cumulative Percent
EXP-MATEMÁTICAS	0	10	30.303	30.303	30.303
	1	12	36.364	36.364	66.667
	2	11	33.333	33.333	100.000
	Missing	0	0.000		
	Total	33	100.000		
EXP-DANZA	0	9	50.000	50.000	50.000
	1	5	27.778	27.778	77.778
	2	4	22.222	22.222	100.000
	Missing	0	0.000		
	Total	18	100.000		
CONTROL	0	10	31.250	31.250	31.250
	1	16	50.000	50.000	81.250
	2	6	18.750	18.750	100.000
	Missing	0	0.000		
	Total	32	100.000		

As can be observed, no student is located at level 3 in any of the groups under investigation. At level 1, a similar distribution is recorded across the three groups: 34.4%, 34.4%, and 31%. At level 2, the highest percentage is occupied by the control group with 48.4%, the experimental mathematics group occupies 36%, and 15% corresponds to the experimental dance group. Finally, at level 2, the highest percentage is held by the experimental mathematics group with 52.3%.

Figure 1

Pretest Results Van Hiele Geometric Reasoning.



3.5 POSTTEST RESULTS

Table 9 shows that in the posttest, out of the 33 participating students in the experimental mathematics group, 5 (15%) were located at level 0, while 18% were located at level 1. At level 2 of reasoning, 54.5% were found, and at level 3, 4 students appeared in the posttest, representing 12% of the participating students. Similarly, in the experimental dance group, a difference in the percentage of participants is observed compared to the pretest. Only 1 student out of 18, representing 5.5%, was located at level 0 this time, 44% occupied level 1, 38.8% were at level 2, and 11% were located at level 3. These data show a significant change in the experimental groups between the pretest and posttest, which will be statistically validated below.

Table 7

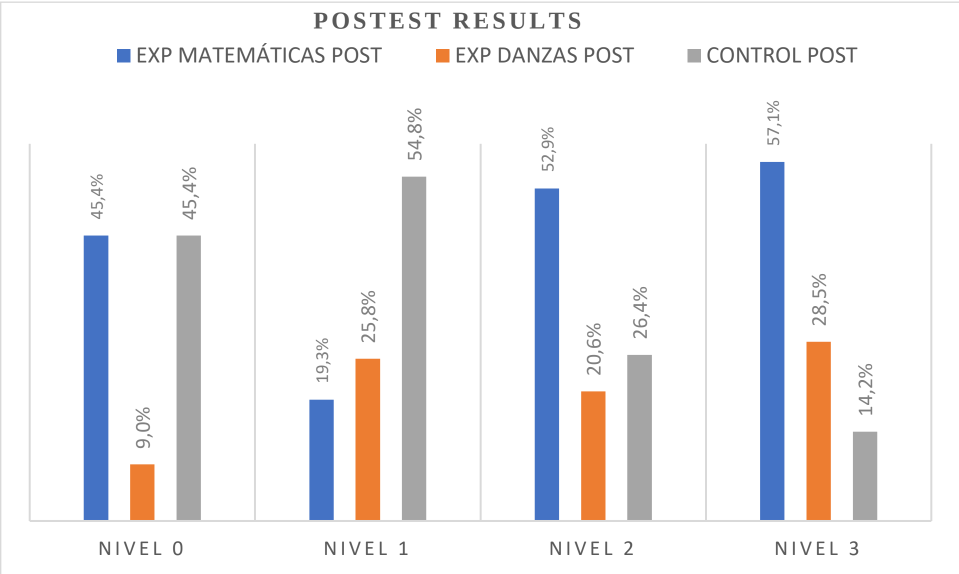
Posttest Results by Groups Van Hiele.

GRUPO	NIVEL POST	Frequency	Percent	Valid Percent	Cumulative Percent
EXP-MATEMÁTICAS	0	5	15.152	15.152	15.152
	1	6	18.182	18.182	33.333
	2	18	54.545	54.545	87.879
	3	4	12.121	12.121	100.000
	Missing	0	0.000		
	Total	33	100.000		
EXP-DANZA	0	1	5.556	5.556	5.556
	1	8	44.444	44.444	50.000
	2	7	38.889	38.889	88.889
	3	2	11.111	11.111	100.000
	Missing	0	0.000		
	Total	18	100.000		
CONTROL	0	5	15.625	15.625	15.625
	1	17	53.125	53.125	68.750
	2	9	28.125	28.125	96.875
	3	1	3.125	3.125	100.000
	Missing	0	0.000		
	Total	32	100.000		

Finally, the control group with 32 students is distributed as follows: at level 0, 5 students, corresponding to 15.6% of the group; at level 1, 17 students, representing 53%; at level 2, 28% of the students; and at level 3, only 1 student (3%).

Figure 2

Posttest Results Van Hiele Geometric Reasoning.



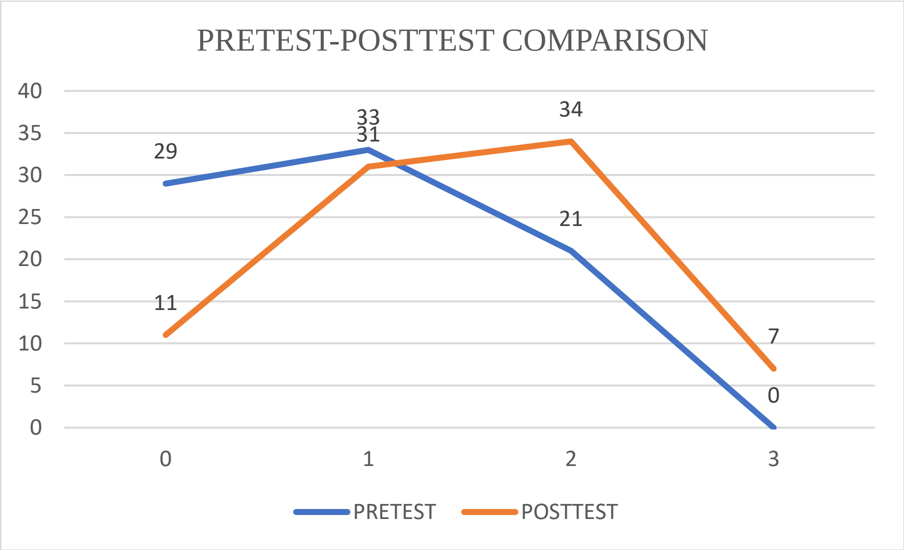
3.6 PRETEST – POSTTEST COMPARISONS

In the pretest, a total of 29 students were located at level 0, while in the posttest this number decreased to 11. This suggests a significant improvement in the basic understanding of geometric concepts. The results show that 33 students were at level 1 in the pretest, and this figure remained relatively stable in the posttest with 31 students. It is important to highlight the need to strengthen development towards higher levels.

Although the study variable is not continuous, Figure 2 shows a frequency polygon resulting from connecting the points corresponding to the percentages of each level for the pretest and posttest. In this way, it can be seen how the two curves intersect between levels 2 and 3, with the pretest curve dropping drastically and the posttest curve increasing notably, clearly showing the trend of the students in the experimental groups to move to higher levels at the end of the implementation.

Figure 3

Pretest – Posttest Comparison Van Hiele Geometric Reasoning.



A notable increase is observed in the number of students located at level 2, rising from 21 in the pretest to 34 in the posttest. This indicates significant progress in geometric reasoning ability. Although no student reached level 3 in the pretest, 7 students were recorded at this more advanced level in the posttest. This result is encouraging and demonstrates substantial progress in geometric thinking.

Figure 4

Pretest – Posttest Comparison of Geometric Reasoning by Groups.

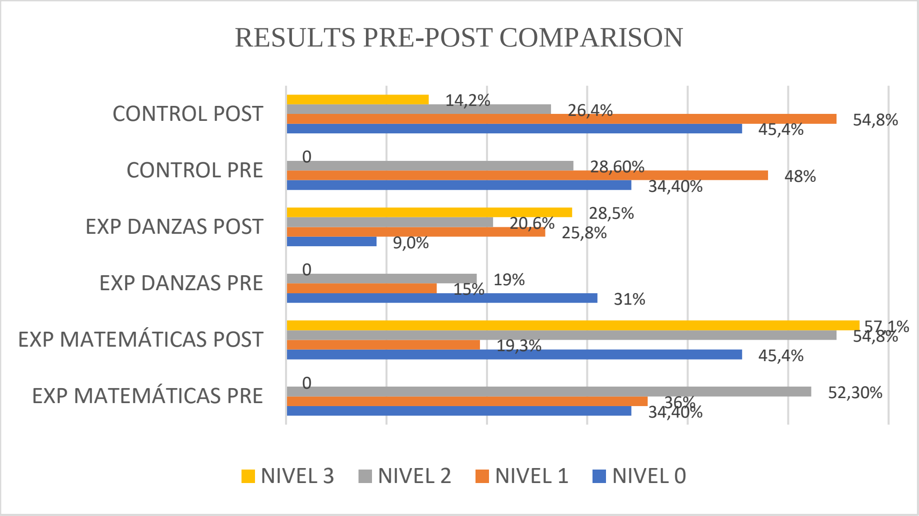


Table 8 identifies the percentage of students who remained at each level after the implementation of the integration proposal for the experimental groups and the traditional class with the same contents as the control group. 13.79% of the students who started at level 0 in the pretest remained at the same level, 48.2% who started at level 0 moved to level 1, 34% ended up at level 2, and 3% advanced to level 3. Of the students who started at level 1, 21% dropped to level 0, 33% remained at the same level, 39% moved up to level 2, and 6% advanced to level 3. For students who started at level 2, 28% dropped to level 1, 52% remained at level 2, and 19% moved up to level 3.

Table 8

Percentage of Students Who Remained at Their Initial Level.

NIVEL PRET	NIVEL POST	Frequency	Percent	Valid Percent	Cumulative Percent
0	0	4	13.793	13.793	13.793
	1	14	48.276	48.276	62.069
	2	10	34.483	34.483	96.552
	3	1	3.448	3.448	100.000
	Missing	0	0.000		
	Total	29	100.000		
1	0	7	21.212	21.212	21.212
	1	11	33.333	33.333	54.545
	2	13	39.394	39.394	93.939
	3	2	6.061	6.061	100.000
	Missing	0	0.000		
	Total	33	100.000		
2	0	0	0.000	0.000	0.000
	1	6	28.571	28.571	28.571
	2	11	52.381	52.381	80.952
	3	4	19.048	19.048	100.000
	Missing	0	0.000		
	Total	21	100.000		

To compare the results obtained in the pretest and posttest, cross tables were prepared with the information collected at both times. Table 9 shows how the students from each level in the pretest were distributed across the various Van Hiele levels in the posttest for each experimental group and the control group.

Table 9

Comparative Pretest-Posttest Results by Group.

		NIVEL POST				
NIVEL PRET	GRUPO	0	1	2	3	Total
0	EXP- MATEMÁTICAS	2	4	4	0	10
	EXP-DANZA	1	6	2	0	9
	CONTROL	1	4	4	1	10
	Total	4	14	10	1	29
1	EXP- MATEMÁTICAS	3	1	6	2	12
	EXP-DANZA	0	2	3	0	5
	CONTROL	4	8	4	0	16
	Total	7	11	13	2	33
2	EXP- MATEMÁTICAS	0	1	8	2	11
	EXP-DANZA	0	0	2	2	4
	CONTROL	0	5	1	0	6
	Total	0	6	11	4	21
Total	EXP- MATEMÁTICAS	5	6	18	4	33
	EXP-DANZA	1	8	7	2	18
	CONTROL	5	17	9	1	32
	Total	11	31	34	7	83

Table 10 summarizes the variations in Van Hiele levels (pretest-posttest) for the students in the experimental mathematics group. The table shows both the number of students and the corresponding percentages at each level of geometric reasoning before and after the intervention. In total, 33 students participated in the experimental mathematics group, with 4 students dropping a level, 11 remaining the same, 12 advancing one level, and 6 advancing two or more levels. These data demonstrate a significant change in the levels of geometric reasoning between the pretest and posttest in the experimental groups, validating the effectiveness of the didactic design based on integrating dance into mathematics teaching.

Table 10

Summary of Variations in Van Hiele Levels for Experimental Mathematics Group.

EXPERIMENTAL MATH GROUP Summary of Variations in Van Hiele Levels (Pretest-Posttest) (Number of Students)

Starting Level	Drops	Remains Same	Moves Up 1 Level	Moves Up 2 or More Levels	Totals
0	0	2	4	4	10
1	3	1	6	2	12
2	1	8	2	0	11
3	0	0	0	0	0
4	0	0	0	0	0
Total	4	11	12	6	33

EXPERIMENTAL MATH GROUP - PERCENTAGES Summary of Variations in Van Hiele Levels (Pretest-Posttest) (Percentage of Students)

Starting Level	Drops	Remains Same	Moves Up 1 Level	Moves Up 2 or More Levels	Totals
0	-	6.06%	12.12%	12.12%	10
1	9.09%	3.03%	18.18%	6.06%	12
2	3.03%	24.24%	6.06%	-	11
3	-	-	-	-	0
4	-	-	-	-	0
% Students	12.12%	33.33%	36.36%	18.18%	100

In the case of the experimental dance group, there were no instances of students decreasing in level, and 13 out of the 18 students moved up a level, which is 72.2%. This indicates that the integrative proposal had a positive effect on this experimental group as well.

Table 11

Summary of Variations in Van Hiele Levels for Control Group.

CONTROL GROUP Summary of Variations in Van Hiele Levels (Pretest-Posttest) (Number of Students)

Starting Level	Drops	Remains Same	Moves Up 1 Level	Moves Up 2 or More Levels	Totals
0	0	1	4	5	10
1	4	8	4	0	16
2	5	1	0	0	6
3	0	0	0	0	0
4	0	0	0	0	0
Total	9	10	8	5	32

CONTROL GROUP - PERCENTAGES Summary of Variations in Van Hiele Levels (Pretest-Posttest) (Percentage of Students)

Starting Level	Drops	Remains Same	Moves Up 1 Level	Moves Up 2 or More Levels	Totals
0	-	3.13%	12.50%	15.63%	10
1	12.50%	25.00%	12.50%	-	16
2	15.63%	3.13%	-	-	6
3	0	0	0	0	0
4	0	0	0	0	0
% Students	28.13%	31.25%	25.00%	15.63%	100

The results in the control group indicate that there were also 9 cases where students dropped a level, which is 28% of the students. 13 students showed an increase in level, and 31% remained the same. While this is a significant percentage, the increase is more noticeable in the experimental groups.

Table 12

Pretest-Posttest Variations.

VARIACIONES PRETEST-POSTEST	GRUPO	N	Mean	SD	SE	Coefficient of variation
PRESTEST	CONTROL	32	0.875	0.707	0.125	0.808
	EXP- MATEMÁTICAS	33	1.030	0.810	0.141	0.786
	EXP-DANZA	18	0.722	0.826	0.195	1.144
POSTEST	CONTROL	32	1.188	0.738	0.130	0.621
	EXP- MATEMÁTICAS	33	1.636	0.895	0.156	0.547
	EXP-DANZA	18	1.556	0.784	0.185	0.504

3.7 STATISTICAL VALIDATION PROCEDURE

This study focuses on evaluating the effectiveness of the intervention designed to improve the understanding of geometric thinking in students by integrating geometry and dance, using a Van Hiele test as the measurement instrument. To achieve this, a statistical analysis was conducted, which included the Wilcoxon test to compare the pretest and posttest results, the Shapiro-Wilk test to verify the normality of the data, and the Point-Biserial Correlation Coefficient (r_pb) to quantify the association between the two variables.

The results of the Shapiro-Wilk test indicated that the data from the scores obtained in the pretest and posttest did not follow a normal distribution (p = 0.001). Subsequently, the Wilcoxon test was applied to compare the pretest and posttest scores, finding a significant difference between them (p = 0.001). Additionally, the effect size was calculated to determine the magnitude of the difference between the groups. An effect size of 4.243 was found, indicating a substantial difference between the groups, which suggests that the intervention had a significant impact on the posttest results compared to the pretest. These findings suggest that the implemented intervention had a significant impact on improving the understanding of geometric thinking in the students.

Table 13

Statistical Validation Results.

Paired Samples T-Test

Paired Samples T-Test

Measure 1		Measure 2	W	z	df	p	Rank-Biserial Correlation	SE Rank-Biserial Correlation
NIVEL PRET	-	NIVEL POST	292.500	−4.243		< .001	−0.646	0.151

Note. Wilcoxon signed-rank test.

Assumption Checks

Test of Normality (Shapiro-Wilk)

			W	p
NIVEL PRET	-	NIVEL POST	0.897	< .001

Note. Significant results suggest a deviation from normality.

4 DISCUSSION

Thus, the main contribution of this work consists of designing and implementing a curricular integration proposal between mathematics and dance. The conclusions presented below, derived from the research work, result from linking three stages.

The first stage involves identifying didactic and epistemological convergences, arising both from exploring their ways of constructing knowledge and from the thinking and reasoning processes of both disciplines. Didactically, the importance of recovering the interaction with the concrete, which the arts bring to mathematics in secondary education, is evident—

particularly in post-pandemic generations—while reasoning through mathematical thinking necessary for choreographic creation and execution. Additionally, dance brings proximity, fun, and enjoyment, creating a socio-affective environment where small achievements, the sensation of discovery, and modeled learning enhance self-efficacy and improve levels of self-confidence in mathematics. Furthermore, one of the main gaps this research has addressed is demonstrating the impact of movement-based intervention in secondary education, giving children and young people the opportunity to reconnect with others and themselves by recognizing their own bodies, emphasizing imagination and creativity as fundamental parts of learning.

From an epistemological perspective, space is dynamically explored as a primary element in mathematical modeling processes and its relationship with the body and movement in dance. The interaction of symbolic representation systems is achieved through the design of floor plans, as well as the search for patterns and structures in the observation and description of choreographic creations. In the exploration and construction of floor plans, the symbolic forms of both areas of knowledge converge while enhancing the dynamic exploration of space and its relationship with the moving subject.

Cognitively, the convergence of concepts is achieved, which, although placed in different contexts, are constructed to nominate similar notions, concepts, and relationships. This similarity allows many elements to be conceptually transferred from one area of knowledge to the other.

The second stage is developed by exploring different perspectives of integration in the Latin American context, which arise from: the perception of dance as a language and a tool for visualizing abstract mathematical concepts, mathematical principles as a solid foundation for choreographic creation, determining that the body can be considered a geometric instrument, and understanding how dance and mathematics are based on the triad of structure, pattern, and organization. These principles, used in the integration design, demonstrated how both areas of knowledge benefit from the results.

The third and final stage involves designing and implementing a series of ten thematic units with students from a public school in Bogotá, with the application and analysis of pretest and posttest to demonstrate the results. From this implementation and corresponding study, it is possible to determine that the intervention of the moving body enhances the development of geometric reasoning in its first three levels, generates higher levels of motivation, and favors students' approach and self-confidence towards choreographic creations. Beyond observing, analyzing, and describing, the socio-affective impact of dance on mathematics is evaluated.

However, it is important to mention the limitations evidenced in the research. Firstly, these integrations transcend conceptions about school mathematical knowledge, challenging the view of mathematics as an exact and infallible science, and suggesting the possibility of seeing it in close contexts, intuitively, culturally, and constructivistly, with the didactic implications that this entails. Moreover, it is a challenge for the dance teacher to rethink the moving body as a protagonist in mathematical constructions that favor precision and accuracy and nourish choreographic creation processes, overcoming their reservations towards this area of knowledge.

The research has revealed a series of significant reflections and learnings that allow us to value and reconfigure the role of the integration between dance and mathematics within the Colombian educational context. The STEAM approach, integrating the disciplines of Science, Technology, Engineering, Art, and Mathematics, has proven to be a valuable framework for transforming the traditional curriculum. This research confirms that the inclusion of dance, as an artistic manifestation, in the teaching of mathematics can significantly increase students' motivation and interest in STEM disciplines. This interdisciplinary approach not only breaks with the fragmented view of the curriculum but also fosters creativity and innovation among students, essential skills for the 21st century.

One of the most relevant findings is the positive impact of dance on the development of geometric thinking. According to the Van Hiele model, students showed notable improvements in the levels of visualization and recognition. It is observed that in the post-test, out of the 33 participating students in the experimental mathematics group, 5 (15%) were located at level 0, while 18% were at level 1. At level 2 of reasoning, 54.5% were found, and at level 3, 4 students appeared, representing 12% of the participants.

Similarly, in the experimental dance group, a significant difference is observed in the percentage of participants compared to the pretest. Only one student out of 18, representing 5.5%, was at level 0 in the post-test, 44% were at level 1, 38.8% were at level 2, and 11% were at level 3. These data show an important change in the experimental groups between the pretest and post-test, validating the hypothesis that dance can be an effective tool for teaching geometric concepts.

Although significant progress in formal deduction was not observed, experimentation through dance facilitated the construction of new geometric relationships and properties. The constant interaction with movements and geometric figures in dance allowed students to internalize and visualize abstract concepts tangibly. These results suggest that the moving body can be an effective tool for teaching geometric concepts, providing a richer and more multisensory learning experience.

5 CONCLUSION

The objective of this research was to address the problem of low performance and lack of motivation towards mathematics, as well as the insufficient recognition of the cognitive contribution of artistic dance education in Colombia for secondary education. Additionally, a curricular solution was proposed under the STEAM approach to enhance geometric reasoning as a cognitive component present in both areas of knowledge, thereby increasing motivation towards mathematics by incorporating the arts. Implementing an integrated curriculum between dance and mathematics presents several challenges. Among them are transforming the rigorous and abstract perception of mathematics into a more intuitive and concrete one; creating didactic spaces that promote spontaneity, autonomy, and experimentation; and positioning the body as a protagonist in the learning process. Despite these challenges, the research offers a unique opportunity to rethink the curriculum and adopt strategies that integrate art and science, providing a more comprehensive and diverse education.

Including dance in mathematics teaching not only enriches learning but also has the potential to generate significant changes in educational policies and curriculum development. Recognizing the importance of the arts in education can lead to curriculums that promote creative, motivated citizens capable of applying knowledge in real contexts. This study highlights the need for greater integration of the arts in education, not as auxiliary disciplines but as fundamental components of comprehensive learning.

In conclusion, this study has demonstrated that integrating dance into mathematics teaching can transform education, enhancing motivation, aesthetic appreciation, and the development of geometric thinking. These findings invite a profound reflection on how the arts can significantly contribute to STEM education, promoting a more holistic and meaningful learning experience for students.

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