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Article

Granule-State Intelligent Mathematics for Analyzing Unseen Risks

Chong-fu Huang [1] and Yun-dong Huang [2,*]

[1] Academy of Disaster Risk Sciences, Faculty of Geographical Science, Beijing Normal University, Beijing (100875), China
[2] The Bill Munday School of Business, St. Edward's University, Austin, Texas (78704), USA
* Correspondence: yhuang2@stedwards.edu

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Abstract: An unseen risk (UR) means that there is miserly information about that risk. Traditional mathematical paradigms for risk analysis, especially probability models based on the law of large numbers, cannot analyze any UR. In this paper, we for the first time propose granule-state intelligent mathematics (GSIM) based on granules and states as basic elements. "Concept", "Knowledge", and "Consciousness" are referred to as granules in thinking activities. A certain situation in which a granule may change is called the state of that granule. Traditional mathematics is a special case of GSIM. All operations and models in traditional mathematics are operations and models in GSIM. In addition, the basic operations in GSIM should at least include averaging, cracking, interaction, stacking, and fusion. This paper discusses the granule-state diffusion model that achieves "drawing inferences about other cases from one instance" through mutual reference and uses this model to analyze again a case of death risk in a virtual city at the beginning of the corona virus disease 2019 (COVID-19) outbreak in March 2020. According to the diffusion model of GSIM, the possibility of "Significant Increase" in infection rate is 0.354, which is much higher than the possibility of "Stable" which is 0.131. It is inferred that within the next 30 days, in this city with a population of 10 million, the death toll of COVID-19 will exceed 189170, the result of analysis in 2020. It has been proven that the infection rate in uncontrolled areas has significantly increased since March 2020. The paper's core contribution is not in the suggested model, but the view of point that it is times to study IM for supporting artificial intelligence (AI). GSIM opened a narrow seam to show a new world, where AI supported by IM might have consciousness and will be smarter.

Keywords: Unseen Risk; Artificial Intelligence; Traditional Mathematics; Intelligent Mathematics; Stream of Consciousness; Granule-State Diffusion; COVID-19

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1. Introduction

Risks that are difficult to analyze using existing methods and obtained data are called unseen risks (URs), and the disasters caused by them are sharpening. From the corona virus disease 2019 (COVID-19) pandemic to the ongoing Russia-Ukraine, people are repeatedly reminded that, faced with URs, humans cannot gamble their lives. Developing mechanical artificial intelligence (AI) into AI with risk awareness may help people more accurately capture and govern URs, greatly reducing survival costs. Even if statistical inference, machine learning, and stochastic computing perform well, with the continuous breakthrough of computing power and the emergence of artificial general intelligence (AGI) that can simulate the physical world, it will still be mechanical AI, because the current theories and methods are essentially prepared for mechanical AI.

The currently popular AI based on deep learning algorithms, called generative AI, whose underlying model is an artificial neural network (ANN). Using a certain learning method, an ANN diffuses the statistical information from the given learning samples onto the connections among neurons in weight values, and forms classification thresholds. A trained ANN can be well used for pattern recognition. Deep learning refers to that an ANN has many hidden layers and can learn big data more deeply. This type of AI that only has the ability to "parrot the tongue" and does not think of any new problems, and does not have real intelligence, is referred to as mechanical AI in this paper. It cannot analyze URs because there is no big data available for learning.

Mechanical AI is rapidly accelerating technological and social change. AlphaFold is able to predict 98.5% human protein structure, Google DeepMind's AI has discovered 2.2 million new materials, and OpenAI's GPT-4o has ability to handle text, speech, and video, creating conditions for interaction between us and machines.AI has advantages such as "comprehensiveness of knowledge", "not knowing fatigue" and "fast speed". If we can make it conscious and generate some wisdom, it can help us analyze URs.

AI holds the promise of tremendous benefits for humanity, also accompanies emerging risks. AI could be used to engineer new pandemics or for propaganda, censorship, and surveillance, or released to autonomously pursue harmful goals. AI run amok that could lead to nuclear catastrophe. However, these AI risks are not fundamentally different from those brought about by cars, appliances, and nuclear devices. When we study the risk issues of AI, we should focus more on how to enable AI to analyze URs. Only when AI can analyze its own URs towards humans and avoid them, can safety be guaranteed. If we passively study AI risks, either we are bound by our hands and feet, or we become increasingly passive and unable to prevent them.

AI is an ancient industry, and the superpowers of modern computers enable them to quickly complete existing algorithms, such as large-scale artificial neural networks, thus achieving commercialization. In fact, today's AI only has the statistical learning and decision optimization capabilities supported by traditional mathematics (TM), without real intelligence [1]. Numbers and space are fundamental elements in TM, used to describe the physical world rather than human intelligence.

Mathematics that describes consciousness and takes human thinking activities as research field is called intelligent mathematics (IM) [2]. Concept, knowledge, and consciousness are the fundamental elements of IM. Obviously, without IM, it is impossible to generate conscious AI. It is also impossible to analyze URs using AI. There is no big data available for learning, no matter how computing power and large model are excellent; the mechanical AI is also ignorant of URs.

The intelligence in artificial intelligence refers to the ability to understand, analyze, and solve problems, which is obtained through learning and training. This ability includes logical thinking, mathematical ability, language ability, production ability and survival ability. Intelligence is the ability to process and apply information. The intelligence in intelligent mathematics involves human wisdom, which is the judgment, insight, and wise decision-making ability formed on the basis of instinct, knowledge, experience, and understanding. It is a comprehensive application of values and moral principles, surpassing simple knowledge and skills. Instinct is an innate impulse or action motivation, an uneducated behavior pattern determined by genes. Instinctive wisdom is an innate, intrinsic, and non-learning or experiential behavioral pattern that is a fixed response mechanism within living organisms, used to ensure the survival and reproduction of species.

"Drawing inferences about other cases from one instance" is one of the instinctive wisdom of humans to deal with the unknown world. This wisdom lies in the ability to see a bigger picture with incomplete knowledge, so that giving us a premonition of URs. However, due to limited knowledge, fatigue, and slow response time of individuals and teams, it is difficult for them to capture and govern URs in a timely manner. Such inferences and premonition are streams of consciousness process, not simply statistical learning and decision optimization, as URs do not provide big data. "Drawing inferences about other cases from one instance" is a diffusion extrapolation, i.e. inferring a series of possibilities from a small amount of information.

In 1997, this phenomenon is described as the principle of information diffusion [3]: When we use incomplete data to estimate a causal relationship, there must exist reasonable diffusion functions and corresponding operation to partly fill the gaps in the incompleteness, making diffusion estimation closer to the true relationship than non-diffusion estimation. The diffusion function is interpretable and more flexible, so widely used in risk analysis.

At the beginning of the COVID-19 outbreak, there was very little information about the disease, and its death risk is unseen. The traditional mathematical paradigms for risk analysis, especially probability models based on the law of large numbers, cannot analyze this risk. In March 2020, the author analyzed a virtual case using the information diffusion technology (IDT) to process incomplete information on an AI platform called "Internet of Intelligence" (IOI). The results show that the studied city with ten million people, in the next 30 days, the death toll to COVID-19 might be 189170.

IDT can also be used to process flexible information in IOI to assess dynamic risks. It may be argued that IDT would have some distinct advantage to analyze URs but is still in framework of TM. The current IDT cannot model the stream of consciousness, because traditional mathematics cannot describe the consciousness in the human brain. But the principle of information diffusion may be a brick that opens the door to IM, as it has touched upon the expression of "drawing inferences about other cases from one instance" in the stream of consciousness.

If IDT could truly stimulate conscious AI to help people more accurately capture and govern URs, the developing track of the world will change. If all of this is achieved, perhaps there will be no next COVID-19 pandemics, no next large-scale wars, and the planet will become better and better.

Reviewing the limitations of current AI based on TM, in this paper we for the first time propose the granule-state IM, so that the framework of IM proposed by Huang (2018) [2] has a landing point. Employing granule-state IM, we again analyze a case of death risk in a virtual city at the beginning of COVID-19outbreak in March 2020. It shows that granule-state IM not only has the ability to analyze URs, but also gives richer conclusions.

The paper is organized as follows: Section 2 reviews traditional mathematics; Section 3 explores the limitations of current artificial intelligence based on traditional mathematics; Section 4 introduces the framework of intelligent mathematics; Section 5 proposes granule-state intelligence mathematics; Section 6 proposes reference-based granule-state diffusion model; In section 7, we again analyzed the death risk case of a virtual city at the beginning of COVID-19 outbreak in March 2020 to show how to use granule-state intelligent mathematics. We conclude this article with Section 8.

2. Traditional Mathematics

The first generation of mathematics studies static problems, with representative symbols such as Arabic numerals, arithmetic operators, geometric shapes, algebraic symbols, and matrices. People went from simple counting to magnificent algebraic systems and persevered in proving a large number of laws about numbers. After thousands of years, they constructed the first-generation mathematics; the representative symbols of the second-generation mathematics that studies dynamic problems are limits, infinity, infinitesimal, differentiation, integration, vectors, and various spatial symbols. From Newton's need to study the laws of celestial motion, to Cauchy's concise and rigorous proof of Newton Leibniz calculus, and then to the controversial probability theory with respect to subjective or non-subjective, after more than 300 years, people have constructed the second generation mathematics; The third generation of mathematics that studies mathematical structures and generalization problems, with representative symbols such as sets, mappings, and relationships. From Cantor's proposal of set theory to establish the axiomatic foundation of mathematics, to the emergence of a large number of modern mathematics such as topology, fuzzy mathematics, and manifold, after more than 100 years, people have constructed the third-generation mathematics. In this paper, we refer to these three generations of mathematics as traditional mathematics (TM).

The core content of the first generation of mathematics is algebra and geometry. The study of numbers belongs to the category of algebra; the study of shapes belongs to the category of geometry. Algebra is concerned with arithmetic operations using symbols. Algebra helps in the representation of problems or situations as mathematical expressions. It involves variables like x, y, z, and mathematical operations like addition, subtraction, multiplication, and division to form a meaningful mathematical expression. Geometry primarily deals with the shapes and sizes of objects, their relative position, and the properties of space.

“Jiu Jang Suan Shu” (2nd century BC) in China is undoubtedly one of the oldest and most influential works in the first-generation mathematics. “Ying Buzu Shu” is the oldest method for solving algebraic equations [4]. Ancient Greek mathematics represented by Euclid's Elements (3rd century BC) and others established the axiomatic-and-deductive reasoning, i.e., the concept of formal mathematical proof. This innovation draws a clear demarcation between Greek and non-Greek mathematical traditions, exerting an unparalleled influence on subsequent mathematical development [5].

The core content of second-generation mathematics is Calculus. Based on geometric considerations, Newton and Leibniz independently discovered Calculus in the process of introducing a novel way of carrying calculations with variables, whose two major concepts of calculus are Derivatives and Integrals. The derivative gives us the rate of change of a function and the integral gives us the area under the curve. Differential Calculus deals with derivatives, i.e., rates of change and tangents and Integral Calculus deals with integrals, i.e., areas and volumes. In the eighteenth century, as the problems grew in complexity, Calculus indeed became more analytic and since is often described as Mathematical Analysis.

Today, slightly more complex mathematical applications, such as electromagnetic science and AI, cannot do without calculus. In realm of satellite orbital control, whether parameterized in closed form by the Kepler orbital parameters or using Newton's formula to obtain a non-spherical gravity model, the gravity of other celestial bodies and the effect of non-conservative forces such as the propulsion thrust, atmospheric drag and solar radiation pressure [6], acceleration (the derivative of velocity with respect to time) is used. AI is a broad field with no single definition, encompassing research topics that range from symbolic-reasoning-oriented algorithms to cognitive simulation and neuromorphic machines, ultimately leading to artificial neural networks (ANN) [7]. Any training algorithm in ANN, such as the gradient descent method in the backpropagation (BP) networks, is an algorithm for seeking extreme values in Mathematical Analysis [8]. The Riemann geometry used in Einstein's theory of general relativity is a generalization of Gauss' differential geometry [9].

It may be argued that the second generation of mathematics plays the most crucial role in modern science and technology.

The core content of third-generation mathematics is set theory. A set may be thought of as grouping together of single objects into a whole. The existence of sets and basic properties are postulated by the appropriate formal axioms. All mathematical objects can be construed as sets. The formal language of pure set theory allows one to formalize all mathematical notions and arguments. Set theory has become the standard foundation of mathematics.

The properties of geometric shapes that are invariant under any continuous deformation are described using spatial continuity and connectivity on sets, forming general topology, also known as point set topology. With topology as a research tool, Kosterlitz and Thoules showed that superconductivity could occur at low temperatures and also explained the mechanism, a phase transition, that leads to the loss of superconductivity as the temperature rises above a critical point. Meanwhile, Haldane discovered how topology could explain the magnetic properties of certain materials. Nobel Prize in Physics 2016 recognizes their theoretical discoveries of topological phase transitions and topological phases of matter [10]. Zadeh initiated a radically new paradigm by introducing the concept of a fuzzy set, a set in which the membership is a matter of degree rather than a matter of either affirmation or denial [11]. The mathematical models that use fuzzy sets are called fuzzy mathematical models. They are quite different from the crisp models as in crisp models where parameters are fixed which all may not be fixed but may vary due to many reasons. For example, tumors are dynamic in nature leading to dynamic growth patterns, and a fuzzy model can estimate tumor burden according to degree of truthfulness [12]. Using topological homeomorphism, people have extended manifold, a concept of generalized curves and surfaces to more complex arbitrary dimensional objects, resulting in a series of dimensionality reduction algorithms. An n-manifold is a space that is homeomorphic to the n-dimensional Euclidean space. The different dimension reduction algorithms from different manifold homeomorphism constructions have been extensively used in machine learning [13].

Perhaps, mathematics is the language with which God has written the universe. In the hierarchy of scientific disciplines of the Middle Ages and up to the sixteenth century, the mathematical sciences were subordinate to theology and philosophy, and to natural philosophy in particular, but practitioners of mathematical science use TM to tackle physical problems like motion [14]. In other words, TM can only write about the physical universe, but cannot write about all things in the universe, including human intelligence. Nowadays, the numbers and space in traditional mathematics have been generalized, the mathematical system has become extremely complex, and communication among mathematicians has become quite difficult.

The risks that can be recognized by looking back at history may lead to some statistical pattern that can be analyzed using traditional mathematics. However, risks such as COVID-19 and uncontrolled wars are not simply repetitive phenomena in history, but unseen risks that require the emergence of intelligent mathematics to help people analyze. Artificial intelligence is not a part of mathematics, but only an application of mathematical models. Any artificial intelligence model constructed using traditional mathematics lacks the ability to analyze unseen risks.

3. Artificial Intelligence

Before the Wright brothers invented the airplane [15], humans had a long history of attempting to fly in the sky. The legend of artificial intelligence (AI) can be traced back to ancient Egypt, but it can be verified that it originated in the era of the ancient Greek philosopher Aristotle [16] and has a longer history. With the first design of a working computer on paper, corresponding AI has evolved over more than two centuries in a long series of steps [17].

In general, artificial intelligence is considered as the simulation of human intelligence processes by machines. AI is sometimes called machine intelligence and is intelligence demonstrated by machines, in contrast to the natural intelligence displayed by humans and other animals. In computer science AI research is defined as the study of "intelligent agents": any device that perceives its environment and takes actions that maximize its chance of successfully achieving its goals. Colloquially, the term "artificial intelligence" is applied when a machine mimics "cognitive" functions that humans associate with other human minds, such as "learning" and "problem solving". The term "Artificial Intelligence" was coined for the first time by John McCarthy in the Summer Artificial Intelligence Conference, held at Dartmouth College, USA, in 1956 [18].

Modern AI has gone through four stages of development: (1) from Turing machines in 1936 to Zadeh's fuzzy sets in 1965; (2) From fuzzy sets to the theory of information granules in 1997; (3) From information granules to IBM's 2011 supercomputer Watson; (4) From Watson to 2022 ChatGPT developed by OpenAI.

In 1936, Turing published the book On Computable Numbers and proposed an abstract machine, now called a "Turing machine," that moved from one state to another using a precise, finite set of rules (given by a finite table) and, depending on a single symbol, read it from the tape [19]. Today's so-called AI is actually a computer system, and the invention and development of computers is undoubtedly one of the most profound achievements of the 20th century. Turing made three pioneering contributions to it: firstly, to solve the computability problem in mathematics, he developed a simple mathematical model for Turing machine; secondly, he was actively involved in building the first electronic, programmable, digital computers. Finally, he provided an elegant operational definition of thinking that, in many ways, sets the entire field of AI in motion [20]. The first working AI programs were written in 1951 to run on a particular computer, the Ferranti Mark I machine at the University of Manchester (United Kingdom); chequers (i.e., checkers; draughts) – playing program was written by Christopher Strachey and a chess-playing program by Dietrich Prinz [18].

In 1965, Professor Zadeh, as a cybernetic expert, proposed the theory of fuzzy sets and fuzzy logic. His goal is to develop a new path for the development of AI from the mathematical foundation. Classic set theory can only express precise concepts, rigid programs which makes computers not simulate the flexible mobility of human brain in recognition and control due to the use of imprecise (called fuzzy) words. In order to change this situation, he explored the fuzzy concepts through the mathematical description and expanded the classic sets into fuzzy sets [21]. The concept of a grade of membership in fuzzy sets is related to the truth value of a predicate in multivalued logic, but their roles are very different and that the agenda of a theory of fuzzy sets would be very different from the agenda of multivalued logic [22]. This difference promoted a breakthrough to AI, stimulating the emergence of new artificial intelligence technologies such as expert systems [23], knowledge engineering [24], fuzzy control [25], and soft computing [26]. In 1982, the emergence of the Hopfield model [27] brings the artificial neural network (ANN) which was first described by McCulloch and Pitts in 1943 [28] back to life, leading to a research boom in ANN. In response to the fact that much of the evidence on which human decisions are based is both fuzzy and granular, Zadeh proposed the concept of information granules in 1979 [29], and formed the theory of fuzzy information granules in 1997 [30]: There are three basic concepts that underlie human cognition: granulation, organization and causation. Fuzzy information granulation is the concept of a generalized constraint. The principal types of granules are: possibilistic, veristic and probabilistic.

If a century ago, “intelligence” was purely a human’s concept, and human intelligence inspired the researchers to propose the concept of Artificial Intelligence (AI) in the 1960s, it can be said that the emergence of the supercomputer Watson marked the stage of AI's substantial service to humanity, which is the third stage of AI development defined in this article. The origin of Watson dates back to May 1997 when a group of IBM researchers started to develop Deep Blue, an AI solution that can solve complex problems, adapt to the context, and analyze new information. In 2011, IBM introduces the first version of Watson on the popular game show Jeopardy to challenge the reigning champions. It demonstrated how the machine-learning algorithm can search a massive database to find answers to common questions. The outcome was Watson’s victory over human participants [31]. Watson is not connected to the Internet, so it will not search through the network, and only rely on the memory database to answer.

In November 2022, OpenAI, an American artificial intelligence company, published an AI chatbot tool called ChatGPT, marking the fourth stage of AI development. ChatGPT runs on the Internet and the foundation of ChatGPT is Generative Pre-trained Transformer (GPT) architecture [32]. Simply put, ChatGPT is a large language model developed by OpenAI. Its purpose is to generate human-like responses to a wide range of prompts and questions using natural language processing algorithms. Essentially, it is an AI language model designed to simulate human conversation and answer questions to the best of my ability based on the knowledge it has been trained on [33]. Since the release of ChatGPT, numerous studies have highlighted the remarkable performance of ChatGPT, but there is evidence that human programmers maintain a competitive edge over ChatGPT in certain aspects of problem-solving within the programming context. The average score obtained by ChatGPT on the set of IEEExtreme programming problems is 3.9 to 5.8 times lower than the average human score [34].

Obviously, ChatGPT, Sora, Gemini, Claude, and Gork-2 are all powerful AI products today. AI, which uses a large amount of data on the Internet to "understand" the subtleties of human generated natural language, is changing the world at an incredible pace. As an indefatigable helper, AI dramatically changes the world for the better in at least three aspects: transforming transport and urban life, personal assistants, turbocharging scientific and medical research. However, the essence behind the AI mask that relies on machine learning is still statistics. Moreover, these AI programs, despite their widespread applications, often by users who do not understand how they function, are critically flawed. For example, ChatGPT has garnered criticism for, among other things, its tendency to “hallucinate,” a phenomenon in which an AI program responds with factually incorrect answers rendered with a high degree of confidence [17].

It may be argued that the current AI relies on large language models and large training data sets. Unfortunately, the former is still a product of traditional mathematics, such as the ANN model; The latter meant that today's artificial intelligence can only answer questions based on trained data, and any deviation may lead to a series of errors. This kind of AI is actually a smart puppet without any instinctive wisdom. While liberating humans from mechanical affairs, they also have to face many security risk issues [35], let alone expect it to help people capture and govern URs.

The Turing test has provided the inspiration for the inception and rapid development of AI as a discipline. It is perhaps fair to say that no controversy has attained the sweep, scope, or depth of the questions raised by the Turing test. These controversies cover deep philosophical questions on the role and nature of consciousness, spirituality, human intelligence, and the very essence of what it means to be human [36]. From the "thinking machines" of “narrow AI”, the original goal of the AI field, to the artificial general intelligence (AGI) [37] to perform consciously any task that a human can, none of them have surpassed the issues that people were concerned about during the Turing testing period.

The biologically inspired cognitive architecture may be an ideal computational framework for building AGI [38], making it robust, flexible, and adaptable to the environments. It seems we are at a point in time where visibility into naturally intelligent systems is exploding. However, to date, the possibility of an AGI, powerful enough to perform any intellectual task as if it were human, or even improve it, has remained an aspiration, fiction, and considered a risk for our society [39].

The AI supported by traditional mathematical models can do more complex reasoning, more effectively exploit causality, but have not a curiosity to explore new tasks, no moral virtues and other abilities of human being. So, no matter how much computer capability is increased, no matter how complex the models are, it will not evolve into AGI of "human level AI".

Indeed, if AGI achieves superintelligence that greatly exceeds the cognitive performance of humans in virtually all domains of interest, it could pose an existential threat to humanity [40]. For a future “envisioned world" AGI-based uncrewed combat aerial vehicle system, Salmon et al. (2024) [41] identified five main categories of risk: sub-optimal performance risks, goal alignment risks, super-intelligence risks, over-control risks, and enfeeblement risks. They propose to develop controls to manage the risks, including controls on AGI developers, controls within the AGI itself, and broader sociotechnical system controls.

Overall, AGI with superintelligence is still a “null hypothesis” to which the possibility and risks of superintelligence can continue to be ignored [42]. With the current "mathematics" and "0-1" computers, humans can only achieve AGI in the sense of digital intelligence, also known as smart AI, and people must face catastrophic risks. This situation is similar to that if people only followed the idea of simulating a big bird to develop aircraft without aerodynamics, there would not be today's airplanes, and the flight risk would be much higher.

To avoid the catastrophic for humanity caused by superintelligence, Bostrom (2014) [40] posits that society should retard the development of dangerous and harmful technologies, but it is almost hopelessly difficult. We believe that only with the emergence of AI with security awareness can catastrophic risks be effectively avoided. Security awareness is a perception, and what we usually refer to as consciousness is a state. Only those who are in a state of consciousness can perceive safe or unsafe. Consciousness primarily refers to the level of awareness and attention to the external world and oneself. Consciousness is one of the most mysterious phenomena in the brain and the ultimate goal of AI research. We refer to AI that has the ability to perceive the external world and itself as conscious AI, and whether it can reach human-level consciousness is not discussed in this paper.

Reviewing the above four stages of AI development, we venture to predict that that the fifth stage of AI will be an attempt from smart AI to become conscious AI, and a smart puppet will become a wisdom assistant. At that time, limited training data will only play the role of seed numbers, and intelligent mathematical models will enable AI to have security awareness, better serve humanity, and minimize security risks.

4. Intelligent Mathematics

Intelligent Mathematics (IM) is a hot topic, but with a different perspective. Some scholars refer to fuzzy mathematics with lattice structure as IM [43]. There is also a journal named "Intelligent Mathematics" that publishes articles in the fields of mathematics, computer and mechanical technology, economics, humanities, and medical sciences. The author refers to approximate analysis as IM and published a monograph in Springer titled "Intelligent Mathematics: Computational Analysis" [45]. This type of IM is actually a part of traditional mathematics. The IM discussed in this paper is not of this type.

The main goal of AI is to enable machines to complete complex tasks which usually require human intelligence, and it may be the discipline that requires IM the most. However, research on AI is still trapped in models built using algebra and calculus. It is just that, with the rapid development of computer science, the large language models established by traditional mathematics have been able to run, and AI training on big datasets has been achieved. It seems that the ability of AI is increasing exponentially, so that, people mistakenly believe that AI has made a breakthrough.

Mechanism-based AI theory, which unifies structuralism-based AI, functionalism-based AI, and behaviorism-based AI [46], is essentially a theoretical system for processing information based on probability theory, matrices, and mathematical mappings. It still falls under the category of mechanism theory. The generalized logic theory, which aims to support this AI theory [47], treats generalized probability theory as the theoretical basis for intelligent information processing, expanding the existing logical systems for handling uncertainty and calling it a flexible information processing model.

In fact, any mathematics based on algebra and calculus is not a tool for describing human thinking activities and therefore cannot be considered intelligent mathematics. This is because when consciousness is lost following coma, thinking activity stops, which means that the human thinking activities are based on consciousness, and not all consciousness can find a counterpart in the physical world. For example, newborns know how to eat food, driven by survival instincts without any conceptual understanding. Traditional mathematics does not provide tools to describe consciousness, making it impossible to describe instinctual consciousness and thinking activities governed by acquired meaning. Therefore, this article defines intelligent mathematics as follows:

Definition 1: Mathematics that describes consciousness and takes human thinking activities as its research object is called intelligent mathematics.

Consciousness, usually considered a psychological term, primarily refers to the level of awareness and attention to the external world and oneself. In philosophy, consciousness is considered the reflection of the objective material world in the human mind, encompassing sensory, cognitive, and various psychological processes. In clinical medicine, consciousness is concretized as the ability to perceive, understand, and respond to the surrounding environment, one's condition, the relationships between objects in the environment, and the interrelationships between oneself and the environment. In mechanistic AI theory, consciousness is regarded as the ability to perceive, understand, and respond to stimuli from the external and internal environments under certain knowledge support and goal orientation.

Clearly, neither the "original consciousness" in psychology, which considers survival desires, nor the AI-focused "consciousness," which involves knowledge-based information perception, can establish intelligent mathematics.

In psychology, the original consciousness of the research object still has counterparts in the physical world. For example, the original consciousness of "group desire" has its counterpart in the physical world as a "set."

The current AI's focus on "consciousness" also cannot detach from the physical world since its relied "in-formation" refers to the "state and changes" of objects in the material world.

This article argues that the world of consciousness and the physical world are entirely different realms without a mathematically homomorphic relationship. The world of consciousness can partially perceive the physical world, and the physical world can partially perceive the world of consciousness, but most objects in the world of consciousness do not have counterparts in the physical world. For instance, the instinctual consciousness of survival does not mirror any life-death process in the physical world because they operate on entirely different levels.

In human thought exchanges, people often use the physical world as a medium, using metaphors and examples to express their consciousness. However, not all conscious content can be conveyed through this medium. Much of the content in consciousness is ineffable and can only be intuitively understood.

The world of consciousness is not a product of idealism but an objective existence that cannot be fully understood using viewpoints from the physical world.

Consciousness is humanity's final stronghold for understanding the universe and oneself, and it is the core and hope for seeking to transcend genetic limitations and optimize our species. Clearly, a machine without consciousness cannot achieve the level of intelligence that arises from conscious thought, no matter how advanced its automation.

Consciousness, as an object in the non-material world, is still largely mysterious to humans. At present, any attempt to universally define it is futile.

In the territory of traditional mathematics, numbers, operators, and structures are essential. As mathematics has evolved, numbers have been abstracted into variables represented by letters, operators have developed into complex mathematical-physical models, and real-space structures have generalized into sets and even super-topologies.

Perhaps starting from "concept," "knowledge," and "consciousness", we can gradually develop the territory of IM.

4.1. Definition of "Concept"

"Concept" refers to the sensory and perceptual understanding of the essence or whole of an object, whose synonyms include "Idea", "Thought", "Notion" and "Impression", whereas "Notion" is a vague overall impression, especially of something considered incorrect. Established concepts related to old phenomena, clear mathematical concepts, etc., typically use "Concept." For new phenomena or newly formed vague concepts, "Notion" is used more often.

Clearly, the "concept" in cognitive activities is far more complex than those in mathematical logic and cannot all be expressed through intension and extension. The solution cannot be achieved solely by imposing constraints on intension and extension.

From the perspective of intelligent mathematics research, this article defines "concept" as follows:

Definition 2: In the process of cognition, humans abstract and generalize the common essential characteristics of perceived objects, thus elevating perceptual knowledge to rational understanding. This form of expression is called a concept.

For example, commodities, machinery, humans, communication, high, happiness, epidemic, and war are all concepts. It must be emphasized that Definition 2 is not a universal definition but is proposed specifically for studying IM.

4.2. Definition of "Knowledge"

"Knowledge" refers to the understanding formed through extensive perception of a certain type of thing, whose synonyms include “Understanding”, “Comprehension”, “Grasp” and “Grip”. "Understanding" focuses on knowing or perceiving, while "Comprehension" implies grasping or capturing.

In philosophy, the study of knowledge is called epistemology. Over two thousand years ago, the Greek philosopher Plato defined knowledge as "justified true belief." Over three hundred years ago, the British philosopher Bacon proposed the slogan "knowledge is power." Today, the definition of knowledge remains a topic of ongoing debate among philosophers. Plato's three criteria for knowledge are considered insufficient, with additional conditions such as "tracking the truth" being proposed [48].

In psychology, information acquired by individuals through interaction with the environment is called knowledge. Psychologists divide knowledge into declarative knowledge and procedural knowledge. Declarative knowledge describes the characteristics and relationships of objective things and is divided into three levels: symbolic representation, concept, and proposition. Procedural knowledge is a set of steps for doing things, primarily used to solve "what to do" and "how to do it" problems.

Knowledge still lacks a unified and clear definition, but there are some basic consensuses: knowing and understanding is called knowledge. In other words, the structured understanding formed through the perception and recognition of things can be called knowledge. Based on this, this article defines "knowledge" as follows:

Definition 3: The structured understanding formed through extensive perception of a certain type of thing, which has guiding significance, is called knowledge.

For example, "the earth orbits the sun", "the real estate bubble cannot keep expanding indefinitely" and "buildings in areas prone to earthquakes above 6 degrees must be fortified" are all examples of knowledge. Some knowledge is simple, e.g., "1+1=2", while some knowledge is very complex, e.g., "theory of relativity".

There are many types of knowledge. In knowledge engineering alone, there are at least eight methods: predicate logic, production rule, frame, script, process, semantic network, Petri nets, and object-oriented method.

4.3. Definition of "Consciousness"

As mentioned in the previous section, human understanding of the mysteries of "consciousness" is minimal, making it challenging to provide a universal definition.

However, the rigor of mathematics requires the delineation of research objects. From the perspective of IM research, this article defines consciousness as follows:

Definition 4: The basis for controlling one's behavior formed in the human brain through instincts, experiences, and knowledge is called consciousness.

For example, the basis for understanding risk phenomena and governing the risks is called risk consciousness. Consciousness can be divided into instinctual consciousness and acquired consciousness.

Any behavior not dependent on experience is called instinct. Newly hatched sea turtles instinctively move toward the ocean. Instinctual behavior is also called innate behavior. The basis for controlling one's behavior formed in the human brain through instincts is called Instinctive Consciousness or Innate Consciousness.

Instinctual behavior is not necessarily controlled by the brain. Instinct is not the same as instinctual consciousness. Only when awakened does it become consciousness. For example, when a baby is hungry and cries, it indicates that the instinctual consciousness for survival has awakened. If a baby is born with a certain brain dysfunction, the survival instinct may still be there, but the instinctual consciousness of "crying when hungry" may not awaken.

The basis for controlling one's behavior formed in the human brain through experience and knowledge is called Acquired Consciousness.

For example, a student who feels hungry during an exam refrain from eating due to experiential consciousness; when facing the delicious yet poisonous pufferfish, the consciousness formed by knowledge of its toxicity forces the diner to think twice before eating.

In terms of individual experiential consciousness, its content constantly changes and is very complex. To summarize the process by which certain information, emotions, and desires from the external world or the internal subconsciousness continuously move in and out of consciousness, people have created the term "Stream of Consciousness", to represent a chain of instantaneous conscious “now” moments [49]. In the Orch OR (Orchestrated Objective Reduction) theory view, the different state of consciousness (e.g.: normal, altered, dreaming, anesthesia, etc.) could be expressed by stream of consciousness with different physical quantities (such as different frequency, etc.) [50].

The human brain 'dreaming' is considered to be a flowing process of the stream of consciousness. Based on the different contents of the stream of consciousness, it can be divided into information consciousness stream, emotional consciousness stream, and desire consciousness stream. According to the different sources and forms of the stream of consciousness, it can be further classified into internal absorption stream, internal intrusion stream, external absorption stream, and external intrusion stream.

Subconsciousness, which integrates psychology and brain science and is also the most difficult part to get hold of, has a subtle influence on psychological activities in the process of graphic creativity [51]. In daily life, we are surrounded by a bewildering array of signals, some of which are perceived and processed subconsciously [52]. Different from the ‘’unconscious’’ in the Freudian sense, subconsciousness refers to the consciousness that exists in our mind, but we do not realize it. The results of subconscious thinking may be detected when the mind transits to the state corresponding to its concentration on this subject of the thinking [53]. The motive of the self might be the core of the subconsciousness [54]. Emotional function may be a part of the subconscious. The ‘’subconscious’’ is, therefore, permanently in a position to sort through the mass of information we continuously receive, working as a sort of ‘’living autopilot’’. These subconscious behaviors are sufficient to ensure the survival of humans [55].

It is particularly noteworthy that the fluid in the material world is clearly not the original image of the stream of consciousness. The fluid mechanics model cannot describe the stream of consciousness.

In real life and scientific exploration, people use observation and thinking to recognize the unknown world of society and even nature, and the stream of consciousness plays a crucial role in thinking activities. Instinctual consciousness that does not go through brain thinking and acquired consciousness are both components of the stream of consciousness. The components in the stream of consciousness and the information obtained through observation influence the results of thinking activities.

In current AI research, training a complex digital neural network to navigate unfamiliar and dark environments is considered a high level of “intelligence”, but it is entirely different from the creative thinking supported by the stream of consciousness.

In the stream of consciousness, instinctual consciousness is the guardian of life, and acquired consciousness contains the motive for creative thinking.

Knowledge is the soil for the growth of acquired consciousness, and the awakened consciousness of the human brain is the seed. Without the seed, acquired consciousness cannot grow in the soil. Machines that store vast amounts of knowledge cannot generate acquired consciousness without the seed of consciousness and cannot perform creative thinking.

On the face of it, creative thinking seems like a skill to think out of the box [56]. In fact, creative thinking is the performance of creativity in completing a series of creative tasks, and creativity is what drives innovation and progress. The Torrance Tests of Creative Thinking (TTCT) is the most used in educational research and practice including gifted identification [57]. TTCT and other extensive research mainly involve creative thinking at the lowest and middle levels [58]. Even short-term sports activities such as ascending stair-climbing can enhance this creative thinking ability [59]. Creative thinking at the highest level involves personal traits and even genes. With creative thinking at the highest level, Einstein proved that space and time were not absolute but relative to each other and that mass could also be translated into energy. For those who have an inherent desire for knowledge, the seeds for creative thinking often give them higher creative potential [60]. Fundamentally, scientific research is an extended endeavor of creativity [56].

5. Granule-State Intelligent Mathematics

Based on the description of consciousness and focusing on human thinking activities as the research object, intelligent mathematics involves mathematical description issues ranging from concept generation to the entire process of idea exchange. Establishing such a mathematical system that is completely different from traditional mathematics is a daunting task. This article proposes granule-state intelligent mathematics (GSIM), attempting to go the first step.

In the second stage of AI development mentioned earlier, a granule involved in granular computing [30] is a set on a given universe of discourse. We believe that any individual concept, knowledge, and consciousness are granules. The granules that do not require a certain set to describe are more common.

Definition 5： "Concept", "Knowledge", and "Consciousness" are referred to as granules in thinking activities.

For example, "Most Swedes are tall" is a fuzzy information granule about Swedes' knowledge, which can be defined by a fuzzy set on the universe of discourse of height. For another example, "China Railway High-speed", "Russia-Ukraine conflict", etc. are granules of the "concept" category, and it is unnecessary to use sets to define them. Moreover, "economic environment is improving", "group desire", "rainstorm is coming" and so on are granules of "consciousness", which cannot be defined by sets. That is to say, the granules in GSIM include specific granules defined on sets, but there are more granules that are not constrained by set theory.

In the thinking process of the human brain, various granules with respect to concepts, knowledge, and consciousness flow, which do not require any set or mapping to support them, let alone the meaning of mathematical structures such as topological spaces. Otherwise, it is difficult to explain how wisdom exists in the world of the blind, let alone how ancient people made decisions before the emergence of mathematics. Today, we use mathematical models to simulate a tiny bit of human intelligence with extremely low efficiency, relying entirely on computing unparalleled performance. If a number is considered as a point in zero-dimensional space, then in the vast majority of cases, a granule should be a line, a plane, a solid and a cube in 1, 2, 3, and n-dimensional space, respectively. Human wisdom is more developed on lines, planes, solids, and does not necessarily need to be reduced to a point in order to be expressed clearly. Knowledge such as "A tree height of 100 meters must have a root, and a river flowing thousands of miles must have a source" does not need dimensionality reduction to pixels for explaining. We believe that the development from the digital age to the granule age is an inevitable trend. If functions and sets are treated as granules, then functional analysis [61] and random set theory [62] are both studies of mathematical structures on granular spaces. Fuzzy granular computing is much simpler than them. The "granule" we define today has directly broken free from all previous constraints and is expected to play a key role in intelligent mathematics.

The granules are the fundamental element of thinking, but they must be activated in order to form a stream of consciousness. For example, an infectious disease specialist assessment of the COVID-19 infection rate is a granule. This granule usually is a type of "knowledge" or "consciousness", or it may be a synthesis. However, only if a specialist considers the change of the infection rate with various environments, he could form a subsequent stream of consciousness related to this granule. We refer to the possible changing state of a granule as its state.

Definition 6: A certain situation in which a granule may change is called the state of that granule.

For example, "The average height of Swedes is basically fixed" is a state of knowledge-based fuzzy information granule: "Most Swedes are tall". For another example, "China Railway High-speed is going abroad" is a state of concept-based granule: "China Railway High-speed", while "It is raining three kilometers away" is a state of consciousness-based granule: "Rainstorm is coming". That is to say, a state is an additional dynamic attribute of the granule, which helps the granules to enter the thought engine. Simultaneously capturing granules and states is the intellectual ability of humans to quickly analyze the environment, adapt to the environment, and then improve the environment.

The description of the situation may vary greatly depending on the perspective of observation or thinking. For example, when analyzing the risk of death from infectious diseases, the judgment of the infection rate is a granule, and the possible change of the rate is a state. From the perspective of medical resources, the possibility of the change would be low; Observing or thinking from the perspective of high personnel flow on transportation arteries, the possibility might be high; Describing from the perspective of integrating all influencing factors leads to another state. The perspective of observation or thinking is called a baseplate of the state. Describing the situation of a granule on the same baseplate is beneficial for ideological communication.

Definition 7: The perspective of observing or thinking about a state is called the baseplate of the state.

For example, the baseplate for the state "the average height of Swedes is basically fixed" is "macro perspective", and the baseplate for "China Railway High-speed is going abroad" is "large-scale construction", while the baseplate of "It is raining three kilometers away" is "visual distance ". Sometimes, it is necessary to indicate the baseplate of the state to avoid confusion. In most cases, the baseplate is in some self-evident consensus sense, and it is unnecessary to indicate. For example, it is necessary to indicate whether the baseplate of the state "urban expansion" is land or something else. The baseplate of the "income increase" is self-evident, that is money.

In the stream of consciousness, granules and states are not necessarily quantified. Formally, we denote a granule and a state as g and s, respectively. The evidence composed of them is denoted as:

$$e=g+s@, \tag{1}$$

where, symbol "+" is not addition, but refers to s being attached to g. In email addresses, symbol "@" serves as a separator between username and email server domain name, and also specifies the server to receive emails. In expression (1), symbol "@" means that the state s has dynamism and information communicability.

Just as the introduction of imaginary number i has enabled people to establish the theory of complex analysis and methods, greatly expanding mathematical analysis capabilities, then the introduction of state s could enable us to establish an intelligent mathematics, called Granule-State Intelligent Mathematics (GSIM).

Just as all operations and models in real analysis are available in complex analysis, as a part of GSIM, all operations and models in traditional mathematics are available in GSIM. For example, the addition, subtraction, multiplication, and division of numbers are operations in GSIM; The model of computing with words [63], that uses words to replace numbers for calculation and inference, achieving fuzzy inference and control of complex information systems, is a model in GSIM.

5.1. Properties of Granule

Let,

$$⌖=\{♋,\ \beta,\ \delta\}, \tag{2}$$

where

$$\begin{cases} \alpha \triangleq \text{"Concept"}, \\ \beta \triangleq \text{"Knowledge"}, \\ \delta \triangleq \text{"Consiousness"}. \end{cases} \tag{3}$$

where $\triangleq$ is the definition symbol.

✂●⧹⌖, if the granule g is about ●, it is called a ● category granule and is denoted as g_●. For example, $h \triangleq$ "China Railway High-speed" is a ♋ category granule, denoted as h_♋. Obviously, a photo of a cup and a weighing of a bag of rice are of the same category as ♋. "The cup can be used for drinking water" and "Southerners are accustomed to eating rice" are of the same category as ♌. "Group desire" and "rainstorm is coming" are of the same category as ♎.

Everything described by granules is called an object. For example, physical objects, processes, mechanisms, architectures, and universes in the physical world, numbers, sets, complex analysis, deep learning models in mathematics, and GDP (Gross Domestic Product), CPI (Gross Domestic Product), exchange rates in economics are all objects. COVID-19, War and God are also objects. In fact, they are all objects of human thought.

Definition 8: The manifestation of a granule is called its shape.

For example, the shapes of "China Railway High-speed", "Most Swedes are tall", and "group desire" are all descriptions in words. And a photo of a cup, its shape is a photo. The shapes of infection rates are numbers. The shape of "It is raining three kilometers away" is distance and rainfall. There are countless granules in thinking, both concrete and abstract shapes. Liquid, solid, and gas are common shapes, and objects imagined out of thin air also have shapes. There are both low dimensional shapes, high-dimensional shapes, and chaotic shapes.

Numbers, relatively simple, have already formed four basic operations in production practice: addition, subtraction, multiplication, and division, and have developed differential and integral operations in the eighteenth century. Sets are relatively complex, and whether it is operations at the set level such as union and intersection, or various operations in topological spaces, their computing power has not broken through, because most of them borrow the functions of number operations (such as differentiability of manifolds), and even only borrow names (such as additive group and quotient groups in group theory). That is to say, although all operations and models in traditional mathematics are operations and models in GSIM, the basic operations that can be used in GSIM are only arithmetic operations and calculus. The basic operations in GSIM should at least include averaging, cracking, interaction, stacking, and fusion.

a) **Average:** Generate a new granule-state in the average sense similar to a number. When we say the sum of two numbers, we refer to the total number obtained by adding these two numbers. Dividing this total by 2 is the average of the two numbers. Addition and division constitute the average operation. The average of complex numbers can be calculated by multiplying all complex numbers and dividing them by the number of complex numbers. Due to the different categories, objects, and shapes of granules, as well as the different baseplates of states, it is impossible to have a unified granule-state averaging operation. In the next subsection, we propose an average operation for fuzzy granule-states.

b) **Cracking:** One granule-state cracking produces two granule-states. The simplest cracking: $e=g+s@$ cracking to produce $e_1=g$, $e_2=s@$.

c) **Interaction:** The granule-states of different objects generate new granule-states, e.g., the interaction between people and vehicles when crossing the road. When vehicles first appeared, the interaction between people and vehicles "operate" traffic police and traffic rules. The majority of granular states in IM are non-quantitative. The traffic rules generated by the interaction between people and vehicles are mostly not quantified. For example, at intersections with zebra crossings but no traffic lights, pedestrians go first. There is no need to quantify this rule.

d) **Stacking:** Organizing granule-states according to certain rules. For example, storing a large number of granule-states according to their properties into a granule-state base.

e) **Fusion:** Running multiple granule-states to generate inference results.

The paper's core contribution is not in the suggested model, but the view of point that it is times to study IM for supporting artificial intelligence (AI). GSIM opened a narrow seam to show a new world, where AI supported by IM might have consciousness and will be smarter, so that it can help us to analyze URs.

5.2. Average Operation for Fuzzy Granule-States

Definition 9: A granule-state whose granule and state are expressed by using fuzzy sets is called a fuzzy granule-state.

For example, an infection rate judgment of the percent of patients to COVID-19 in Eq. (4), given by an infectious disease specialist, is a fuzzy granule. The judgment in Eq. (5) of the trend of infectiousness given by the specialist based on his experience is a fuzzy state.

$$g=\left\{\frac{0.1}{0\%},\frac{1}{10\%},\frac{0}{20\%},\frac{0}{30\%},\frac{0}{40\%},\frac{0}{50\%},\frac{0}{60\%},\frac{0}{70\%},\frac{0}{80\%},\frac{0}{90\%},\frac{0}{100\%}\right\}, \tag{4}$$

$$s=\left\{\frac{0}{\text{Significant Decrease}},\frac{0.2}{\text{Slight Decrease}},\frac{0.5}{\text{Stable}},\frac{0.6}{\text{Slight Increase}},\frac{1}{\text{Significant Increase}}\right\}. \tag{5}$$

Let the sets U and V in Eq. (6) and (7) be the universes of discourse for fuzzy granule g and fuzzy state s, respectively.

$$U=\{u_1,u_2,\cdots,u_n\}, \tag{6}$$

$$V=\{v_1,v_2,\cdots,v_m\}. \tag{7}$$

Then, the fuzzy granule g and fuzzy state s are expressed by the fuzzy sets in Eq. (8) and (9), respectively.

$$A_g=(\mu_g(u_1),\mu_g(u_2),\cdots,\mu_g(u_n)), \tag{8}$$

$$B_s=(\mu_s(v_1),\mu_s(v_2),\cdots,\mu_s(v_m)). \tag{9}$$

where, $\mu_g(u_i)$ and $\mu_s(v_j)$ are the grades of membership of element u_i in g and element v_j in s, respectively.

Let

$$F=\{f_1,f_2,\ldots,f_q\} \tag{10}$$

be a set of q fuzzy granule-states, $f_k=g_k+s_k@$, $k=1,2,\ldots,q$.

$\forall u \in U$, $v \in V$, let

$$\pi_g(u)=\sum_{1\le k\le q}\mu_{g_k}(u)\,, \tag{11}$$

$$\pi_s(v)=\sum_{1\le k\le q}\mu_{s_k}(v) \tag{12}$$

and let

$$\Pi_F=\begin{cases}(\pi_g(u_1),\pi_g(u_2),\cdots,\pi_g(u_n))\\(\pi_s(v_1),\pi_s(v_2),\cdots,\pi_s(v_m))\end{cases} \tag{13}$$

be the sum of the fuzzy granule-states in F. Then, the average granule and average state of F are given by Eq. (14) and (15), respectively.

$$\overline{A}_g=(\frac{\pi_g(u_1)}{q},\frac{\pi_g(u_2)}{q},\cdots,\frac{\pi_g(u_n)}{q})\,, \tag{14}$$

$$\overline{B}_s=(\frac{\pi_s(v_1)}{q},\frac{\pi_s(v_2)}{q},\cdots,\frac{\pi_s(v_m)}{q})\,. \tag{15}$$

6. Reference-based Granule-State Diffusion Model

The key to human wisdom surpassing mechanical artificial intelligence is not memory, not speed, and certainly not endurance, but the ability to inference about other cases from one instance. This ability involves association and diffusion processing to granule-states in the stream of consciousness. This section proposes a reference-based granule-state diffusion model to partially achieve this ability. Unlike the traditional information diffusion technique [64], the suggested granule-state diffusion is not performed through any diffusion function, so that, theoretically the model can be applied to any type of granule-states.

Let

$$E=\{e_1,e_2,\ldots.e_q\} \tag{16}$$

be a set of q granule-states, $e_k=g_k+s_k@$, $k=1, 2, \ldots, q$, and different granules can be compared in terms of similarity, and the same goes for states. Unlike set F in Eq. (10), g and s in E are not necessarily fuzzy sets.

The symbols er_g and er_s represent certain methods of comparing granules and states, respectively. Traditional information diffusion technique that partially compensates for information gaps involves diffusing observations around, which requires observations to have a range. General granules and states, having broken free from the constraints of traditional mathematics, lack the concept of range. Only when granule-states are objects in traditional mathematics can distance be defined and traditional information diffusion technique can be used. For a general set of granule-states E, we can only use a symbol er to represent the method of comparing similarity.

For any granule-state, the human brain has methods of comparison, most of which are not mathematical formulas. For example, when comparing two beauties, an ordinary person will not use mathematical methods to measure their similarity, and in most cases, it depends on personal preference and can change with the environment.

The "drawing inferences about other cases from one instance" in the stream of consciousness usually occurs within a person's brain but often incorporates others' suggestions to adjust the granule-states. We systematize this model and propose a reference-based granule-state diffusion model to achieve "drawing inferences about other cases from one instance".

Without loss of generality, we assume that the number of granule-states that can be referenced is q>4, and assume that

$$\mathscr{er}_g\ (g_1, g_2),\ \mathscr{er}_g\ (g_1, g_3),\ \mathscr{er}_g\ (g_1, g_4) > \mathscr{er}_g\ (g_1, g_k),\ k \geq 4. \tag{17}$$

Then, we draw inferences s_2, s_3, s_4 from s_1. That is to say, under the assumption Eq. (17), s_2, s_3, s_4 diffuse some to s_1. The average of s_1, s_2, s_3, s_4 is the adjusted state corresponding to granule g_1. With respect to the granule-state diffusion to achieve "drawing inferences about other cases from one instance", "three cases" is the simplest function. Therefore, we assume q>4, and three granules or states can be selected to adjust the instance when it is not absolutely trustworthy.

It should be noted that, for the convenience of writing, we assume that g_2, g_3, g_4 are closest to g_1; Other cases follow similarly with slightly more complex subscripts.

The above method of "drawing inferences about other cases from one instance" to diffuse states based on the similarity between granules can also be used to diffuse granules. Without loss of generality, we assume

$$\mathscr{er}_s(s_1, s_{k1}),\ \mathscr{er}_s(s_1, s_{k2}),\ \mathscr{er}_s(s_1, s_{k3}) > \mathscr{er}_s(s_1, s_k),\ k \neq k1, k2, k3. \tag{18}$$

Then g_{k1}, g_{k2}, g_{k3} diffuse some to g_1. Their average is the adjusted granule corresponding to s_1.

When E is a set of fuzzy granule-states, we use the lattice degree of nearness proposed by Wang (1980) [65] as a similarity, denoted by L.

Definition 10: Let A and B be two fuzzy sets on U, denoted as

$$A \circ B = \underset{u \in U}{\vee} (A(u) \wedge B(u)), \tag{19}$$

$$A \odot B = \underset{u \in U}{\wedge} (A(u) \vee B(u)), \tag{20}$$

are called the inner product and outer product of A and B, respectively. Here, $\vee, \wedge$ denote the maximum and minimum values, respectively.

$$L(A,B) = \frac{1}{2}[(A \circ B + (1 - A \odot B)] \tag{21}$$

is called the lattice degree of nearness between A and B.

For example, let $U=\{a, b, c, d, e, f\}$,

$$A=0.6/a+0.8/b+1/c+0.8/d+0.6/e+0.4/f,$$
$$B=0.4/a+0.6/b+0.8/c+1/d+0.8/e+0.6/f.$$

Then

$$\begin{aligned} A \circ B &= (0.6 \wedge 0.4) \vee (0.8 \wedge 0.6) \vee (1 \wedge 0.8) \vee (0.8 \wedge 1) \vee (0.6 \wedge 0.8) \vee (0.4 \wedge 0.6) \\ &= 0.4 \vee 0.6 \vee 0.8 \vee 0.8 \vee 0.6 \vee 0.4 \\ &= 0.8, \end{aligned}$$

$$\begin{aligned} A \odot B &= (0.6 \vee 0.4) \wedge (0.8 \vee 0.6) \wedge (1 \vee 0.8) \wedge (0.8 \vee \mathbf{1}) \wedge (0.6 \vee 0.8) \wedge (0.4 \vee 0.6) \\ &= 0.6 \wedge 0.8 \wedge 1 \wedge 1 \wedge 0.8 \wedge 0.6 \\ &= 0.6, \end{aligned}$$

$$L(A, B) = \frac{1}{2}[A \circ B + (1 - A \odot B)]$$

$$= \frac{1}{2}[0.8+(1-0.6)]$$

$$= 0.6.$$

That is, the lattice degree of nearness between A and B is 0.6. The larger the value of L, the more similar the two are. It is particularly important to note that when the maximum grade of membership in a fuzzy set is less than 1, the lattice degree of nearness of the fuzzy set itself is also less than 1. Since the reference-based granule-state diffusion in this paper only requires comparison of relative similarities, the intuitive and convenient lattice degree of nearness is chosen.

7. Analyzing Unseen Death Risk of COVID-19 Using Granule-State Intelligent Mathematics

In 2024, it is easy to write a story about 2019 coronavirus disease (COVID-19), because the three-year-long pandemic ended in 2022, and a large amount of evidence is enough to enrich the story. However, before and at the beginning of the outbreak, it was difficult to analyze death risk of COVID-19 due to insufficient evidence.

Reviewing the beginning of the outbreak of COVID-19, we have the following facts:

- December 2019, multiple cases of viral pneumonia were discovered in Wuhan.
- On January 12, 2020, the World Health Organization named this new virus "2019 novel coronavirus“.
- January 18, 2020, Wuhan Baibuting Community held a "Ten Thousand Family Banquet“.
- January 20, 2020, the authorities confirmed that the suspected case is infective.
- January 23, 2020, the authorities placed a lockdown in Wuhan from 10:00 am.
- February 11, 2020, the World Health Organization named the disease "coronavirus disease 2019 " (COVID-19).

In March 2020, the death risk of COVID-19 was a typical unseen risk (UR). The reference [66] proposed a hybrid model supported by the internet of intelligence to analyze a death risk under incomplete information. The network that connects intelligent agents with computers is called the Internet of Intelligence (IOI) [67], where an individual who can provide message, experience, knowledge, and judgment to a customer to solve problems is called an intelligent agent. In IOI that is used by the reference [66], infectious disease specialists and local doctors are intelligent agents, who integrate collected evidence and the experiences of known infectious diseases to provide personal judgments on the death risk of COVID-19. In this section, we use the virtual case of this article to study how to use granule-state intelligence mathematics (GSIM) to analyze the risk of COVID-19 mortality.

In this case, there are ten million people in the studied city. The IOI asked about fuzzy judgments of COVID-19 infection and death rates in the city to infer how many people will die of COVID-19 in the next 30 days. Ten infectious disease specialists and eight local doctors provided personal judgments on IOI, that are similar the information collected from the Internet during December 8, 2019- January 10, 2020 [66]. Based on the judgments of these 18 people, which are regarded as 18 granules, we can roughly infer their judgments on the development of the epidemic by analyzing their optimism and nervousness, thus forming 18 states.

7.1. Fuzzy Granule-State of Infection Rate

We use Eq. (22) and (23) to represent the universe of discourse for the granule and state of infection rates, respectively. The fuzzy granular-states of infection rates are shown in Table 1, e.g., Specialist 5 (IDS5) believes that the possibility of 10% of people being infected is 0.5, and the possibility of COVID-19 being slight decrease within the next 30 days is 0.6.

$$\begin{aligned} U &= \{u_1, u_2, \cdots, u_{11}\} \\ &= \{0\%, 10\%, 20\%, 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, 90\%, 100\%\} \end{aligned} \tag{22}$$

$$\begin{aligned} V &= \{v_1, v_2, \cdots, v_5\} \\ &= \{\text{Significant Decrease, Slight Decrease, Stable, Slight Increase, Significant Increase}\} \end{aligned} \tag{23}$$

Table 1. The fuzzy judgments of the percent of patients who might be infected by the COVID-19 virus in the studied city in the next 30 days, and fuzzy judgments of possible changes in the infection rate.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	v_1	v_2	v_3	v_4	v_5
IDS1	0.1	1	0	0	0	0	0	0	0	0	0	0.0	0.2	0.5	0.6	1.0
IDS2	0	0.8	1	0.1	0	0	0	0	0	0	0	0.0	0.0	0.0	0.8	1.0
IDS3	0.5	1	0.6	0	0	0	0	0	0	0	0	0.8	0.4	0.2	0.1	0.0
IDS4	0.2	1	0.1	0	0	0	0	0	0	0	0	0.8	0.3	0.1	0.0	0.0
IDS5	1	0.5	0	0	0	0	0	0	0	0	0	1.0	0.6	0.0	0.0	0.0
IDS6	0.3	0.2	0.1	0	0	0	0	0	0	0	0	1.0	0.8	0.0	0.0	0.0
IDS7	0	0.4	0.8	1	0.7	0.6	0.2	0.1	0	0	0	0.0	0.0	0.0	0.6	1.0
IDS8	0	0.9	1	0.7	0.5	0	0	0	0	0	0	0.0	0.0	0.0	0.7	0.8
IDS9	0.4	0.5	0.7	0.8	0.9	1	0.9	0.8	0.7	0.1	0	0.0	0.0	0.0	0.0	1.0
IDS10	0	0.8	0.9	1	0.7	0.5	0.1	0	0	0	0	0.0	0.0	0.3	0.4	1.0
LD1	0	0.5	0.6	0.8	1	0.7	0.6	0.2	0.1	0	0	0.0	0.0	0.1	0.7	0.9
LD2	0	0.8	1	0.8	0	0	0	0	0	0	0	0.0	0.2	1.0	0.2	0.0
LD3	0	0.4	0.8	1	0.8	0.4	0	0	0	0	0	0.0	0.0	0.5	0.6	1.0
LD4	0	0	0.3	0.6	0.8	1	0.7	0.2	0.1	0	0	0.0	0.0	0.0	0.8	1.0
LD5	0	0.1	0.4	0.8	1	0.9	0.8	0.4	0.3	0.1	0	0.0	0.0	0.2	0.6	0.8
LD6	0	0	0	0.6	1	0.6	0.3	0	0	0	0	0.0	0.0	0.0	0.9	1.0
LD7	0.2	0.8	1	0.9	0.7	0.6	0.5	0.2	0.1	0	0	0.2	0.5	1.0	0.3	0.1
LD8	0.6	1	0.6	0	0	0	0	0	0	0	0	0.9	0.3	0.2	0.0	0.0

Note: IDS--- infectious disease specialist, LD---local doctor. The values in the table are membership in term of fuzzy set.

For the ith record in the table, we represent the fuzzy granule and fuzzy state with the possibility distributions A_i and B_i, respectively, i=1, 2, ..., 18. For example, for the 7th local doctor (LD7), his fuzzy granule-state is:

$$A_{17}=(0.2,0.8,1,0.9,0.7,0.6,0.5,0.2,0.1,0,0), \qquad B_{17}=(0.2,0.5,1,0.3,0.1)$$

i.e.,

$$e_{17}=A_{17}+B_{17}@.$$

Using Eq. (21), we calculate the lattice degree of nearness of A_{17} to other A, resulting in:

$\{L(A_{17}, A_1), L(A_{17}, A_2), ..., L(A_{17}, A_{16}), L(A_{17}, A_{18})\}$

=\{0.90,1.00,0.90,0.90,0.75,0.60,0.95,1.00,0.90,0.95,0.90, 1.00,0.95,0.85,0.90,0.85,1.00,0.90\}.

From front to back, the three granules who are most similar to A_{17} are A_2, A_8, and A_{12}. This indicates that the fuzzy judgments of IDS2, IDS8 and LD2 are basically the same as LD7. Therefore, LD7 can take the other three individuals' judgments on possible change in infection rate as the diffusion results of "drawing inferences about other cases from one instance" and adjust its own judgment on the possibility of change in infection rate. The simplest adjustment is to use Eq. (12) and (15) to calculate the average state of the following four states:

$$B_2=(0.0, 0.0, 0.0, 0.8, 1.0),$$
$$B_8=(0.0, 0.0, 0.0, 0.7, 0.8),$$
$$B_{12}=(0.0, 0.2, 1.0, 0.2, 0.0),$$
$$B_{17}=(0.2, 0.5, 1.0, 0.3, 0.1).$$

We obtain:

$$\begin{aligned} B_{s_{17}} &= (\mu_{s_{17}}(v_1), \mu_{s_{17}}(v_2), \cdots, \mu_{s_{17}}(v_5)) \\ &= (0.050, 0.175, 0.500, 0.500, 0.475) \end{aligned}$$

This means that based on the lattice degree of nearness of granules, LD7 accepted the state diffusion from IDS2, IDS8, and LD2 and adjusted the state. On the other hand, based on the lattice degree of nearness of states, LD7 accepted the granule diffusion from IDS5, IDS6, and LD2 and adjusted his granule, resulting in:

$$\begin{aligned} A_{g_{17}} &= (\mu_{g_{17}}(u_1), \mu_{g_{17}}(u_2), \cdots, \mu_{g_{17}}(u_{11})) \\ &= (0.375, 0.575, 0.525, 0.425, 0.175, 0.150, 0.125, 0.050, 0.025, 0.000, 0.000), \end{aligned}$$

i.e., with the granule-state diffusion model suggested in section 6, the fuzzy granule-state given by the 7th local doctor becomes a new fuzzy granule-state:

$$e'_{17} = A_{g_{17}} + B_{s_{17}} @$$

We conducted "drawing inferences about other cases from one instance" diffusion and averaging on the 18 granule-states in Table 1, and obtained 18 new granule-states, which are shown in Table 2.

To make each granule A_{g_k} and state B_{s_k} have the same importance, we employ Eq. (24) and (25) to normalize the granules and states in Table 2, which is called sum-based normalization.

$$\pi_{g_k}(u_i) = \frac{\mu_{g_k}(u_i)}{\sum_{1\le t\le 11} \mu_{g_k}(u_t)}, \quad k = 1,2,\cdots,18; \, i = 1,2,\ldots,11. \tag{24}$$

$$\pi_{s_k}(v_j) = \frac{\mu_{s_k}(v_j)}{\sum_{1\le t\le 5} \mu_{s_k}(v_t)}, \quad k = 1,2,\cdots,18; \, j = 1,2,\ldots,5. \tag{25}$$

The normalized granules are expressed by Eq. (26). By averaging them, the probability distribution in Eq. (27) can be obtained, which can be used to estimate the probability distribution of the infection rates.

$$\Pi_{g_k} = (\pi_{g_k}(u_1), \pi_{g_k}(u_2), \cdots, \pi_{g_k}(u_{11})), \quad k = 1,2,\cdots,18. \tag{26}$$

$$\begin{aligned} P &= (p(u_1), p(u_2), \cdots, p(u_{11})) \\ &= (0.118, 0.318, 0.216, 0.126, 0.091, 0.073, 0.035, 0.016, 0.006, 0.001, 0.000) \end{aligned} \tag{27}$$

Averaging the 18 normalized states, we obtain a possibility distribution in Eq. (28), which is an estimator of possible changes in infection rate.

$$s_{\text{infection rate}} = \left\{ \frac{0.175}{\text{Significant Decrease}}, \frac{0.112}{\text{Slight Decrease}}, \frac{0.131}{\text{Stable}}, \frac{0.229}{\text{Slight Increase}}, \frac{0.354}{\text{Significant Increase}} \right\}. \quad (28)$$

P in Eq. (27) is a description of the random uncertainty of the current infection rate and $s_{\text{infection rate}}$ represents the trend of possible changes in the infection rate. They are all calculation results obtained by using the "drawing inferences about other cases from one instance" diffusion. Compared with P' in Eq. (29) obtained by using the non-diffusion model in the reference [66], the probability of low infection rate in P has increased, while the probability of high infection rate has decreased. But $s_{\text{infection rate}}$ indicates that there is a high possibility of an increase in infection rates over time.

$$P' = (0.112, 0.282, 0.184, 0.135, 0.115, 0.084, 0.051, 0.020, 0.013, 0.002, 0). \quad (29)$$

Table 2. New fuzzy judgments generated after granule-state diffusion of the percent of patients who might be infected by the COVID-19 virus in the studied city in the next 30 days, as well as new fuzzy judgments of possible changes in the infection rate.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	v_1	v_2	v_3	v_4	v_5
IDS1	0.050	0.800	0.450	0.275	0.175	0.150	0.050	0.025	0.000	0.000	0.000	0.400	0.275	0.325	0.325	0.500
IDS2	0.025	0.750	0.700	0.300	0.175	0.150	0.050	0.025	0.000	0.000	0.000	0.000	0.050	0.250	0.625	0.700
IDS3	0.550	0.875	0.325	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.600	0.325	0.250	0.200	0.250
IDS4	0.475	0.875	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.600	0.300	0.225	0.175	0.250
IDS5	0.775	0.750	0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.475	0.275	0.175	0.350	0.500
IDS6	0.600	0.675	0.325	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.950	0.650	0.050	0.025	0.000
IDS7	0.025	0.650	0.650	0.525	0.350	0.300	0.100	0.050	0.000	0.000	0.000	0.050	0.125	0.325	0.475	0.775
IDS8	0.025	0.775	0.700	0.450	0.300	0.150	0.050	0.025	0.000	0.000	0.000	0.000	0.050	0.250	0.600	0.650
IDS9	0.125	0.675	0.625	0.475	0.400	0.400	0.275	0.225	0.175	0.025	0.000	0.000	0.000	0.075	0.325	0.925
IDS10	0.025	0.750	0.675	0.525	0.350	0.275	0.075	0.025	0.000	0.000	0.000	0.000	0.000	0.150	0.550	1.000
LD1	0.025	0.675	0.600	0.475	0.425	0.325	0.200	0.075	0.025	0.000	0.000	0.000	0.000	0.100	0.500	0.900
LD2	0.075	0.750	0.700	0.675	0.375	0.250	0.125	0.050	0.025	0.000	0.000	0.000	0.100	0.500	0.475	0.450
LD3	0.025	0.650	0.650	0.525	0.375	0.250	0.050	0.025	0.000	0.000	0.000	0.050	0.125	0.450	0.475	0.775
LD4	0.025	0.550	0.525	0.425	0.375	0.400	0.225	0.075	0.025	0.000	0.000	0.000	0.000	0.075	0.525	0.925
LD5	0.025	0.575	0.550	0.475	0.425	0.375	0.250	0.125	0.075	0.025	0.000	0.000	0.000	0.125	0.475	0.875
LD6	0.025	0.550	0.450	0.425	0.425	0.300	0.125	0.025	0.000	0.000	0.000	0.000	0.000	0.075	0.550	0.925
LD7	0.375	0.575	0.525	0.425	0.175	0.150	0.125	0.050	0.025	0.000	0.000	0.050	0.175	0.500	0.500	0.475
LD8	0.600	0.675	0.325	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.625	0.300	0.250	0.175	0.250

7.2. *Mortality Curve*

The mortality curve, also known as the "death curve", is often used to describe the death rates of a particular species or group at different age stages. For example, the U.S. population mortality curve plots age on the x-axis and death rate on the y-axis [68]. However, the mortality curve is also used to describe the relationship between cancer treatment and COVID-19 mortality [69]. Clearly, mortality is related not only to age but also to many other factors. Any curve that uses a variable of death factors on the x-axis and death rate on the y-axis can be called a mortality curve. Age is just one factor. At this time, the mortality curve represents the vulnerability of the life group to the death factor. When the probability distribution of the occurrence of the death factor for the life group is known, the risk of death in terms of expected value, that is, the number of deaths, can be calculated according to the principles of probabilistic risk analysis. The death risk of an infectious disease is determined by the infection rate and the mortality rate of those infected. The higher the infection rate of a major infectious disease, the more strained medical resources become, and the higher the mortality rate. Therefore, the infection rate is an important death factor. This paper uses the infection rate on the x-axis and the mortality rate on the y-axis to form a mortality curve and estimates the COVID-19 death risk using the probability distribution in Eq. (29). In March 2020, there was neither a large amount of COVID-19 infection rate data nor a large amount of COVID-19 mortality rate data. We asked 10 infectious disease specialists and 8 local doctors in Table 1 to make rough estimates of the mortality curve, as shown in Table 3 [66]. The study in [66], did four years ago during a tense situation, aims to explore an approach to analyze death risk of COVID-19 under incomplete information. During the IOI open, only 10 infectious disease specialists and 8 local doctors provided their judgments with respect to the mortality curve for a virtual city. It has been proven that the epidemic is as severe as they had anticipated, although these judgments are for this virtual city. In [66], we predicted the mortality toll from COVID-19 for the next 30 days rather than the next 10 because there was some information about COVID-19 in the past 3 months on the Internet at that time, and it was too short to predict it for the next 10 days. The value of u in the table is taken from Eq. (22). For example, the estimate $f(u_5)$=0.8 given by IDS1 means that when 40% (u_5=40%) of the population in the study city is infected, 0.8 out of 100 people will die. The mortality curve given by Table 3 is shown in Figure 1.

Table 3. The judgments of the mortality curve $f(u)$ with respect to percent u of patients who might be infected by the COVID-19 virus in the studied city in the next 30 days [66].

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}
IDS1	0	0.10	0.50	0.6	0.8	1	1	1	1	1	1
IDS2	0	0.8	1	1.1	1.5	1.5	1.5	1.5	1.5	1.5	1.5
IDS3	0	0.15	0.55	0.9	1	1.2	1.2	1.2	1.2	1.2	1.2
IDS4	0	0.08	0.3	0.5	0.8	0.9	0.9	0.9	0.9	0.9	0.9
IDS5	0	0.03	0.2	0.4	0.6	0.6	0.6	0.6	0.6	0.6	0.6
IDS6	0	0.02	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
IDS7	0	1.5	2.1	3.5	4.02	4.02	4.02	4.02	4.02	4.02	4.02
IDS8	0	1.9	2.7	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8
IDS9	0	2.5	6.04	6.04	6.04	6.04	6.04	6.04	6.04	6.04	6.04
IDS10	0	2	2.5	3.5	4	4	4	4	4	4	4
LD1	0	2	3	4	5	5	5	5	5	5	5
LD2	0	0.7	0.8	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
LD3	0	1.4	1.8	3	3.5	3.5	3.5	3.5	3.5	3.5	3.5
LD4	0	3	5	5.8	5.8	5.8	5.8	5.8	5.8	5.8	5.8
LD5	0	2.6	4	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1
LD6	0	2.5	3.8	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9
LD7	0	1.2	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4
LD8	0	0.13	0.45	0.8	0.9	1	1	1	1	1	1
Average	**0**	**1.256**	**2.019**	**2.619**	**2.831**	**2.864**	**2.864**	**2.864**	**2.864**	**2.864**	**2.864**

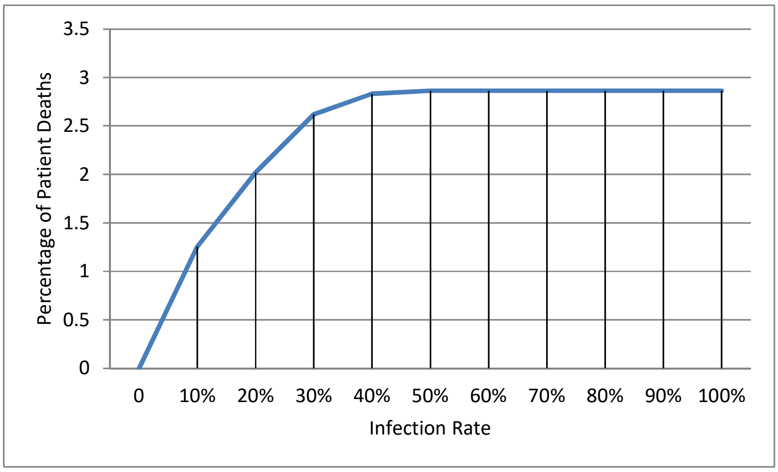

Figure 1. Mortality curve for the studied city with ten million people, in the next 30 days.

Thus, we obtained the discrete mortality curve for COVID-19 in the study city over the next 30 days:

$$\begin{aligned} F(U) &= \{f(u_1), f(u_2), f(u_3), \cdots, f(u_{11})\} \\ &= \{0,1.256,2.019,2.619,2.831,2.864,2.864,2.864,2.864,2.864,2.864\}. \end{aligned}$$

According to the principles of probabilistic risk analysis [70], we infer the death risk of COVID-19,

$$\begin{aligned} D &= \sum_{1\le i\le 11} p(u_i)f(u_i) \\ &= 0.118\times 0 + 0.318\times 1.256 + 0.216\times 2.019 + 0.126\times 2.619 + 0.091\times 2.831 + 0.073\times 2.864 + \\ &\quad 0.035\times 2.864 + 0.016\times 2.864 + 0.006\times 2.864 + 0.001\times 2.864 + 0\times 2.864 \\ &= 1.7983. \end{aligned}$$

Considering there are 10 million population in the studied city, D=1.7983 means that if medical resources and management remain unchanged, the number of deaths from COVID-19 in the next 30 days could be 179,830 (0.017983× 10,000,000 = 179,830). Compared to the death toll of 189,170 predicted in [66], this is 9,340 fewer, but still a very alarming number. The rate infection in Eq. (28) indicates that as time progresses, the likelihood of the infection rate increasing is high, and the number of deaths may be far more than 179,830, enough to attract the attention of the authorities and prompt strong crisis response measures.

Based on the principle of information diffusion [3], the death risk captured by the granule-state diffusion model suggested in this paper is closer to the actual risk than that captured in [66]. Interested readers can use computer simulation experiments to find better methods of "drawing inferences about other cases from one instance" diffusion. The experimental technical route is as follows: (1) Use a computer simulation program to generate a large amount of infection and death data as real data after a certain epidemic ends; (2) Use this large amount of data to calculate the frequency distribution of infection rates and mortality curves. Calculate the expected value of the number of deaths from them, which is regarded as the real risk in the simulation sense; (3) Extract a small amount of data from the simulation data to be the data at the beginning of the epidemic; (4) Generate fuzzy granules and fuzzy states of infection rates for the epidemic by some means referring to this small amount of data; (5) Use N methods to perform "drawing inferences about other cases from one instance" diffusion on the fuzzy granule-states to generate new fuzzy granule-states, respectively; (6) Use the new fuzzy granules to generate N probability distributions of infection rates by N methods; (7) Use these N probability distributions to estimate N death risk values corresponding to the mortality curves in step (2); (8) Calculate the error between these N risk values and the true risk in step (2); (9) The "drawing inferences about other cases from one instance" diffusion method with the smallest error is the best method in one simulation; (10) Randomly select n (no less than 10) seed numbers and perform steps (1)-(9) n times. The diffusion method with the smallest average error is the best method obtained by simulation.

8. Conclusion and Discussion

A risk without sufficient evidence for analysis is an unseen risk (UR). "Drawing inferences about other cases from one instance" is the instinctive wisdom of humans to deal with the unknown world, thus having a premonition of URs. Artificial intelligence (AI) will undoubtedly play an important role in risk analysis, but the current mechanical AI based on traditional mathematics can only analyze the risks familiar to people and cannot analyze URs because there is not enough evidence about URs, so there is no corresponding big data for AI to learn from. Developing conscious AI can help people accurately capture and manage URs, which requires breaking through the constraints of traditional mathematics and establishing AI based on intelligent mathematics (IM).

Whether it is the fantastic, calling the fuzzy mathematics with lattice structure IM; or naming a journal that publishes articles in mathematics, computer and mechanical technology, economics, humanities, and medical sciences "Intelligent Mathematics"; or calling approximate analysis IM, they are actually included in traditional mathematics, because these so-called IMs are not based on describing consciousness and do not take human thinking activities as the research object.

The "concepts," "knowledge," and "consciousness" flowing in the stream of consciousness are the granules in thinking activities. A possible trend of change in a granule is called the state of the granule. IM formed with granules and states as basic elements is called granule-state intelligent mathematics (GSIM). Traditional mathematics, which uses numbers and space as basic elements to describe the physical world, is a special case of GSIM. All operations and models in traditional mathematics are operations and models in GSIM. In addition, GSIM's basic operations should at least include averaging, cracking, interaction, stacking, and fusion.

In the stream of consciousness, granules and states are not necessarily quantified. Most of the granules that vaguely appear in people's minds do not need to be quantified, and a few cannot be quantified. For example, "ghost" cannot be quantified. Ghosts refer to things that vaguely appear, are elusive, and cannot be explained clearly in both the real world and the spiritual world. For example, the phantoms that appear in people's minds are typical "ghosts".

The mutual reference between granule and state in GSIM can achieve "drawing inferences about other cases from one instance" granule-state diffusion. Compared with the information diffusion technology in traditional mathematics, it has greater flexibility and reflects the diversity of association and diffusion in human thinking activities. Reanalyzing a case at the beginning of the COVID-19 outbreak in March 2020 with GSIM, it was inferred that the number of deaths from COVID-19 in the case city in the next 30 days would be far more than 179,830 (calculated by using the 18 granules in Section 7), not just more than 189,170 (result of analysis in 2020 [66]), because the possibility of "Significant Increase" in infection rate is 0.354, which is much higher than the possibility of "Stable" which is 0.131. It has been proven that the infection rate in uncontrolled areas has significantly increased since March 2020.

According to the principle of information diffusion, there are suitable diffusion methods that enable granule-state diffusion to better capture the death risk of COVID-19 at the early stage of the epidemic, providing scientific basis for epidemic prevention and control, so that the global fight against the epidemic does not last for three years. For fuzzy granule-states, the computer simulation experiment is one of the ways to find better diffusion methods. For unquantifiable granule-states, research on diffusion methods is even more challenging.

This paper merely opens a crack in the door for developing IM to assist in risk analysis. It is hoped that people can move beyond the fantasy of "assembling intelligent mathematics from traditional mathematics parts", face the problems like URs that require human wisdom to solve, and promote the development of mechanical AI towards conscious AI, solving some risk analysis problems that traditional mathematical methods cannot solve.

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